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Heating and current drive – magnetically confined plasmas

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Summer Undergraduate Laboratory Internship Summer School

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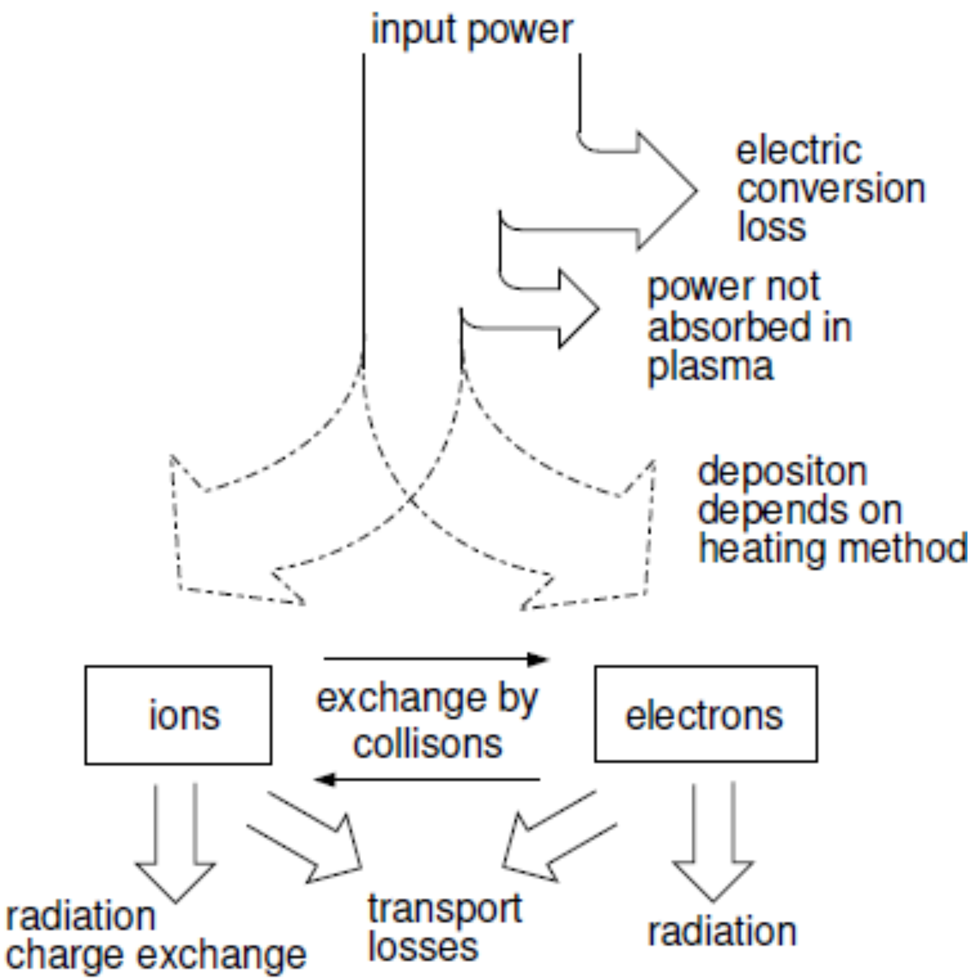
Material from: various S.J. Diem SULI lectures, other talks as noted & J. Coehn NE 536 course at UW-Madison

Current drive and heating required to achieve fusion ignition conditions

$$^{2}D + \ ^{3}T \rightarrow \ ^{4}He(3.5\ MeV) + \ ^{1}n(14.1\ MeV)$$

$$Q = \frac{P_{fusion}}{P_{heat}}$$



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Current drive and heating required to achieve fusion ignition conditions

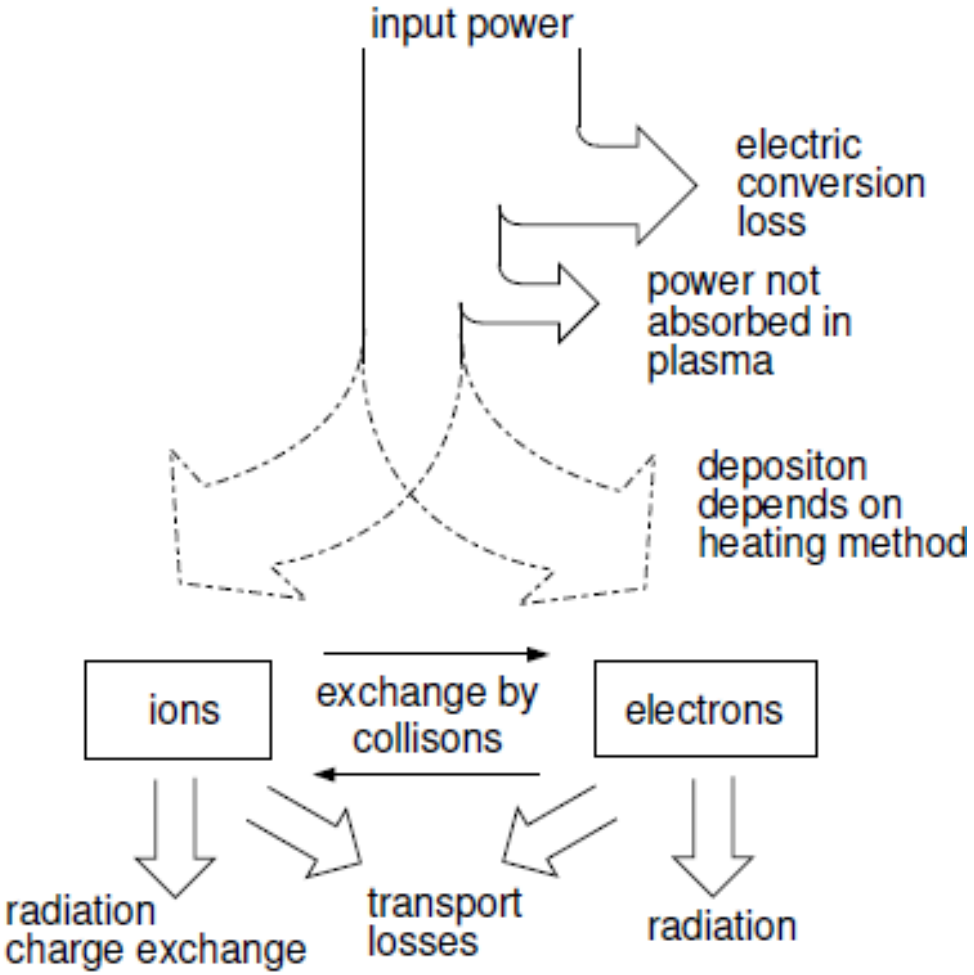
- Reaching ignition requires threshold for:

$$nT\tau_E > 2\text{x}10^{21}\ \text{m}^{-3}\text{-keV-s}$$

~2-3x10^{20} ions/m^3

~100-200 million K

~1-2 seconds



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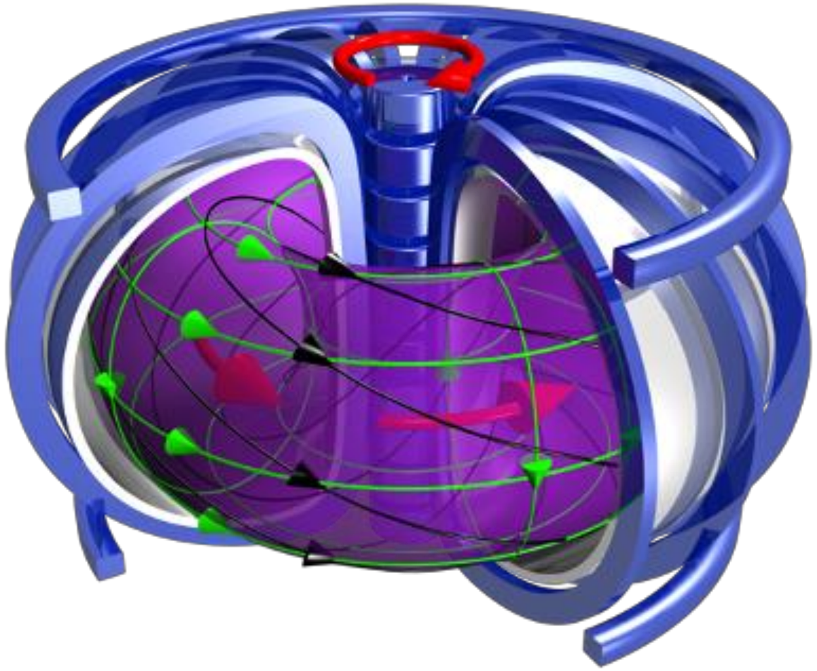

Tokamak: External fields + plasma current to generate confining field

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Plasmas require heating and current drive to achieve fusion conditions

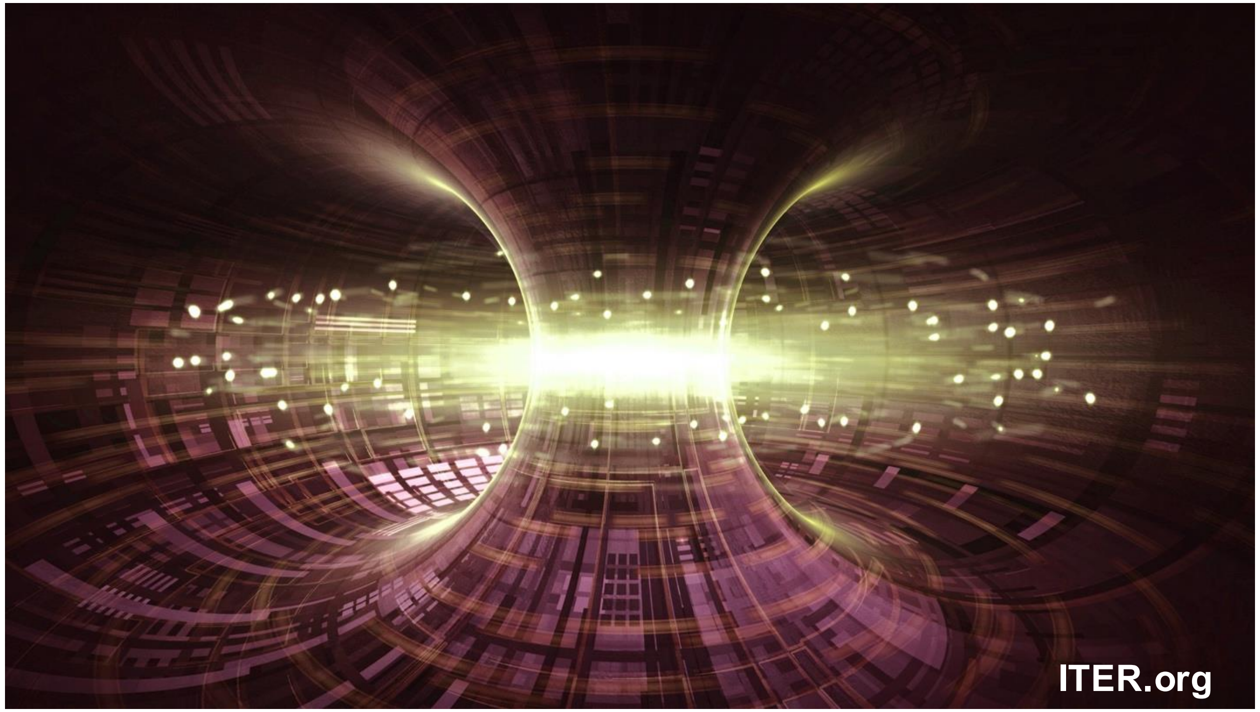
- Reaching ignition requires threshold for: $n \bullet T \bullet \tau_E$
  - External heating required to reach temperature for ignition
  - After ignition, self-heating sustains plasma
- Several methods of external heating & current drive available
  - **Ohmic heating**
  - **Neutral beam injection**
  - **Electron cyclotron resonance heating**
  - Lower hybrid resonance heating
  - Ion cyclotron resonance heating

	Fusion Gain	α-Heating Fraction	Scientific Frontier
	$Q = \frac{P_{fusion}}{P_{heat}}$	$f_\alpha = \frac{P_\alpha}{P_\alpha + P_{heat}}$	
Scientific Breakeven	Q = 1	17%	Alpha confinement
Burning Plasma Regime	Q = 5	50%	Alpha heating; Alpha effects on energetic particle instabilities
	Q = 10	67%	Strong alpha heating; Non-linear coupling effects
	Q = 20	80%	Burn Control; potentially strong non-linear coupling
	Q =	100%	Ignition

From C. Collins

Auxiliary plasma heating required for fusion energy

- Burning plasmas will be heated by alpha particles produced through fusion – but they don't start out that way
- Auxiliary heating power required for: plasma initiation and current ramp up
- Need to heat plasma towards fusion conditions, provide burn control
  - Control MHD instabilities
  - Control against impurity radiation
  - Control plasma profiles
- **Can also provide current drive for long pulse or steady state operation**


ITER.org

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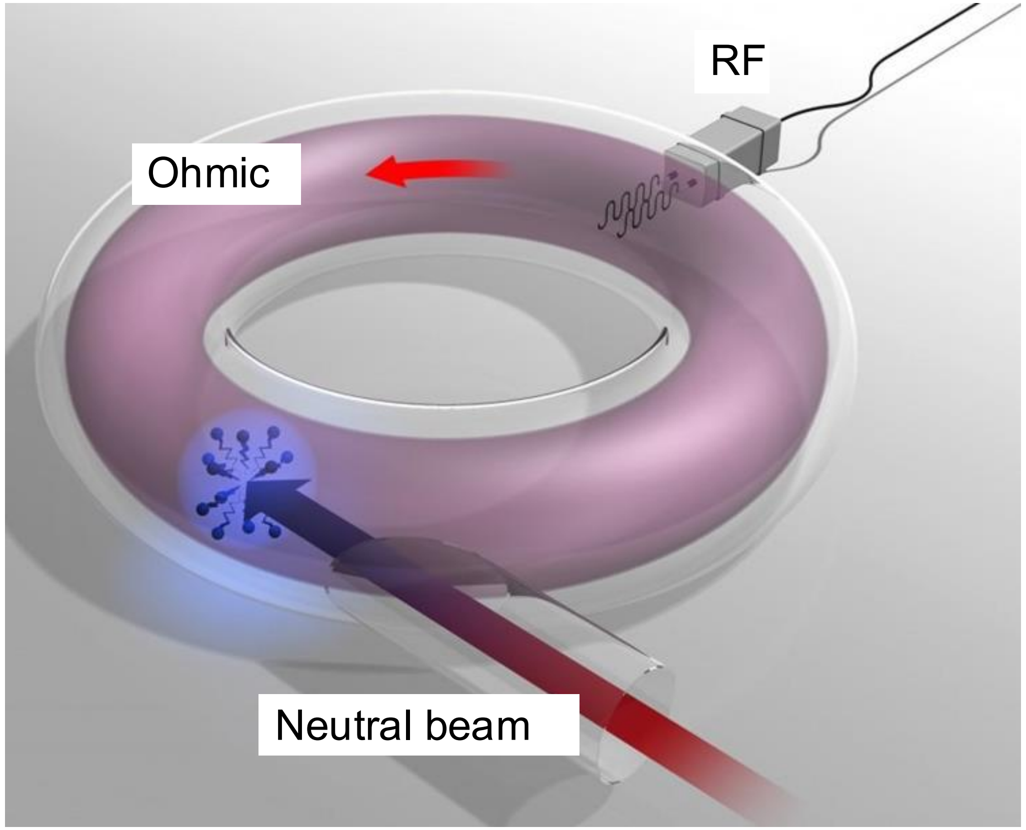

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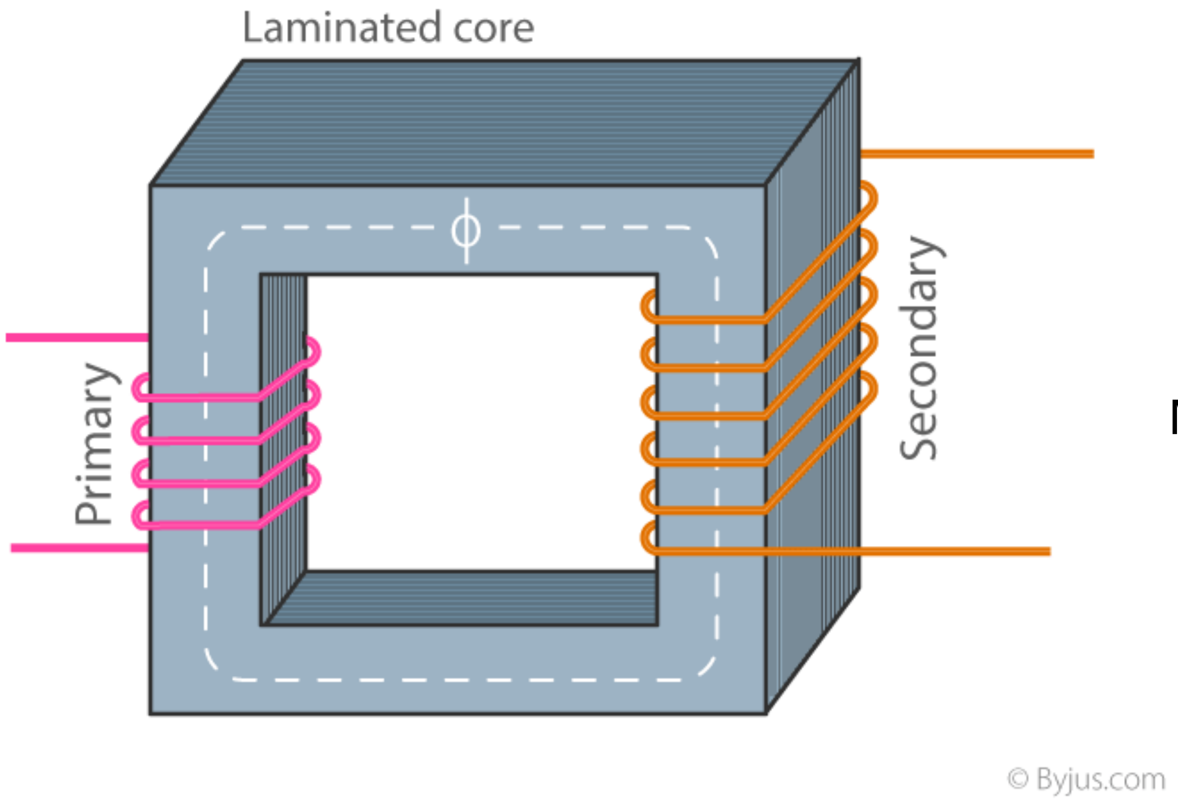
Ohmic heating – like a transformer



Maxell-Faraday equation

$$\oint \bar{E} \cdot d\bar{l} = -\frac{\partial}{\partial t} \int \bar{B} \cdot d\bar{S}$$

Magnetic field of a solenoid

$$B = \frac{\mu_0 \mu_r N}{L} I$$

Toroidal electric field:

$$2\pi R E_\phi = -\frac{\mu_0 N A}{L} \frac{\partial I}{\partial t}$$

Ohmic heating – like a really, really, large transformer

Simplified Ohm's law:

$$\bar{E} + \bar{v} \times \bar{B} = \eta \bar{J}$$

Assume a stationary plasma, take curl of Ohm's law:

$$\nabla \times \bar{E} = -\frac{\partial \bar{B}}{\partial t} = \nabla \times (\eta \bar{J})$$

Take curl, assume η is constant

$$-\frac{\partial}{\partial t} \nabla \times \bar{B} = \eta \nabla \times \nabla \times \bar{J} = \eta [\nabla(\nabla \cdot \bar{J}) - \nabla^2 \bar{J}]$$

Diffusion equation for current:

$$\frac{\partial \bar{J}}{\partial t} = \frac{\eta}{\mu_0} \nabla^2 \bar{J}$$

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Current evolution in tokamaks

For resistivity not a constant, then

$$-\frac{\partial \bar{J}}{\partial t} = \frac{1}{\mu_0}\nabla^2(\eta\bar{J}) - \nabla[\nabla\cdot(\eta\bar{J})]$$

Take toroidal component of the above equation:

$$-\frac{\partial J_\phi}{\partial t} = \frac{1}{\mu_0}\nabla^2\left(\eta\bar{J}_\phi\right)$$

- Current initially driven at surface, then quickly diffuses into plasma
- Resistive heating raises ηJ^2 temperature
- As discharge evolves, current diffuses into plasma core

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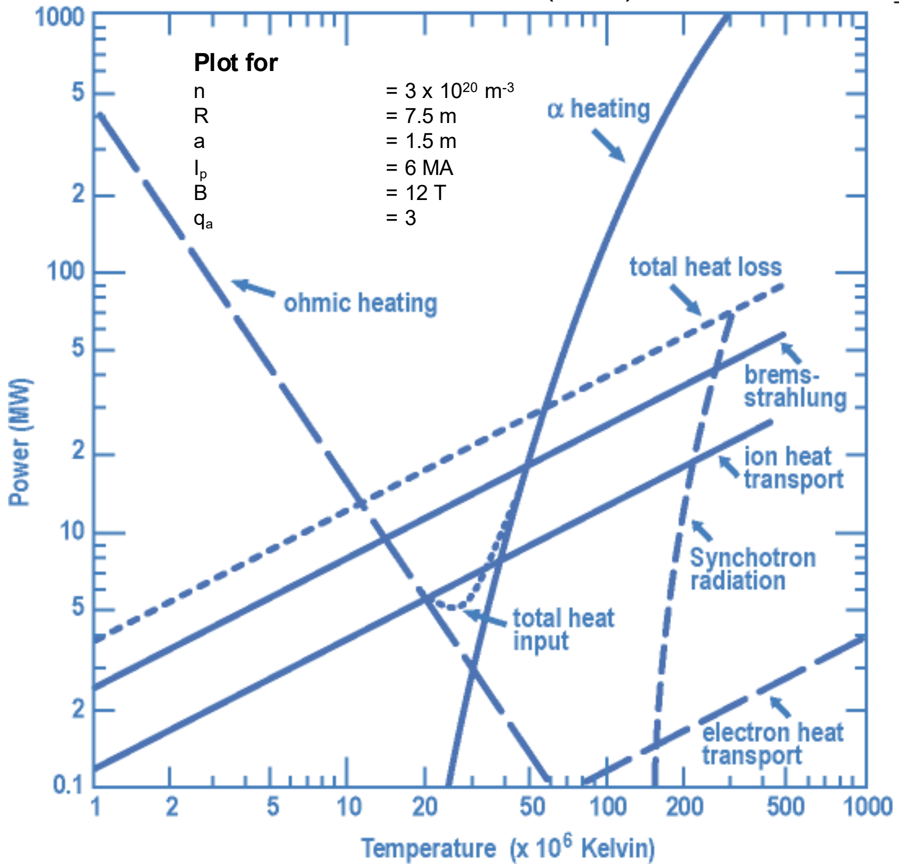
Ohmic heating can only do so much – there's a limit

D.R. Sweetman, Nucl. Fus. 13 (1973) 157-165



Plasma current is limited in tokamaks $j \leq \frac{B_\varphi}{\mu_0 R}$

- Plasma resistivity $\eta \propto T_e^{-3/2}$
- Dissipated power $p = \eta \cdot j^2$
- Plasma stability requires $q_a = \frac{aB_\varphi}{RB_\theta} \geq 2$

Having to balance Ohmic heating with Bremsstrahlung

$$P_{rad,brems}/V = 5.35\times10^{-37} \cdot \left(n_e/m^{-3}\right)^2 \cdot Z_{eff} \cdot \sqrt{T/keV} \qquad \left[W/m^3\right]$$

Means plasma is limited to a few keV

<u>We need auxiliary heating</u>

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Ohmic engineering limits

- Requires time varying current – limits how much current can be driven
- Running current through a solenoid produces heat, limiting how long you can run it
- Ohmic current drive, heating limited to short pulses
  - Cannot be used for long times, steady-state
  - Can be used for plasma startup
- Critical challenge for spherical tokamaks: find a path to operate without an ohmic solenoid
  - Limited space for shielding
  - Drives need for solenoid-free startup

Pegasus-III: Going solenoid free allows more space for TF coils

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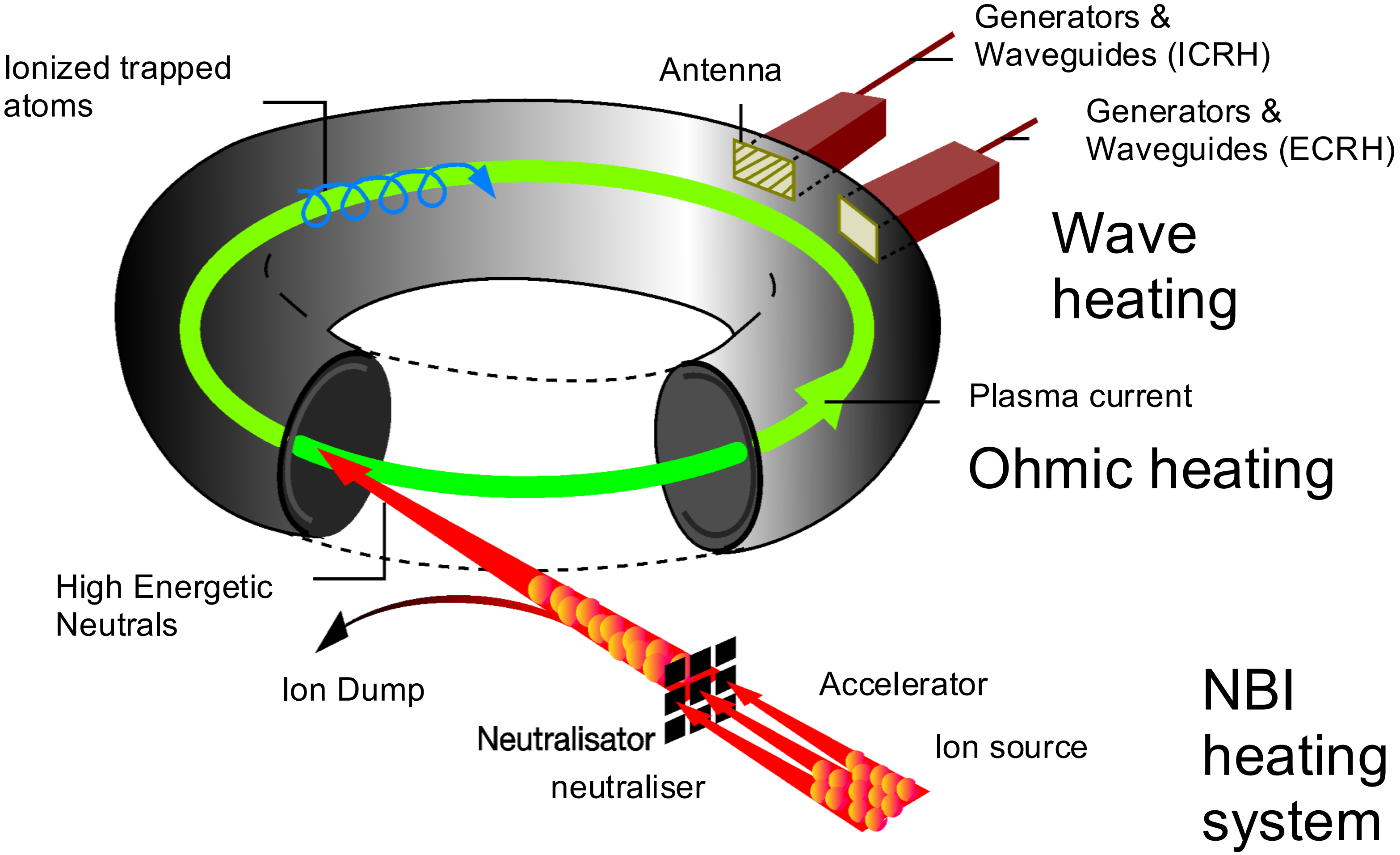
Plasma auxiliary heating

Overview of plasma heating

- Ohmic heating
  - $P = V_{loop} * I_{plasma}$
  - Dissipative heating of current
  - Heats electrons
- Neutral beam injection (NBI)
  - Injection of high energetic neutral fuel atoms into plasma
  - Heating electrons and ions
- Electron cyclotron resonance heating
  - Inject microwaves with $\omega = \omega_{ce}$
  - Heats electrons
- Lower hybrid resonance heating
  - Inject waves with $\omega = (\omega_{ce}\omega_{ci})(1/2)$
  - Heats electrons and ions
- Ion cyclotron resonance heating
  - Injection of radio frequency waves with $\omega = \omega_{ci}$ (10s of MHz)
  - Heats ions



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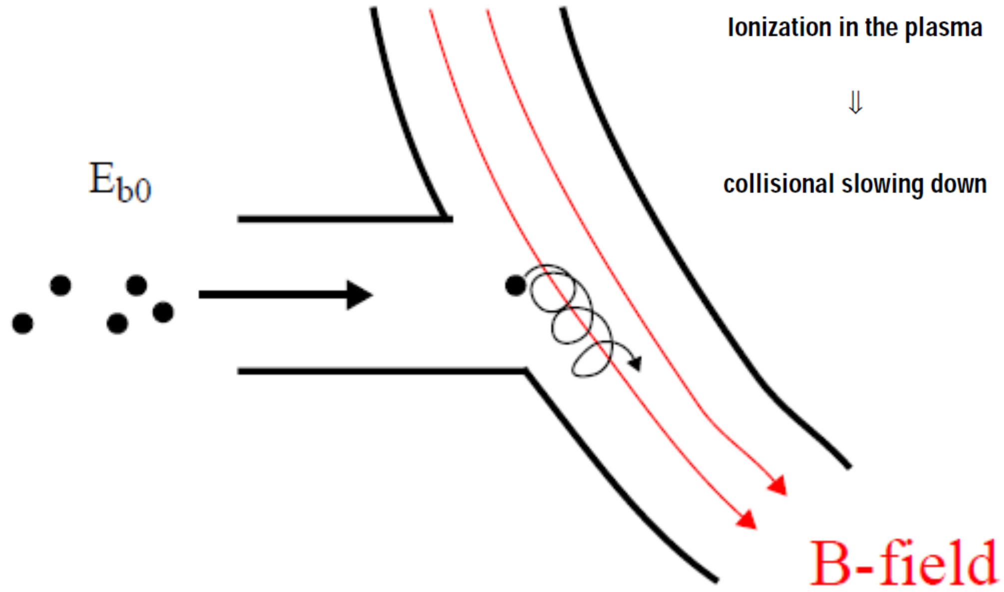
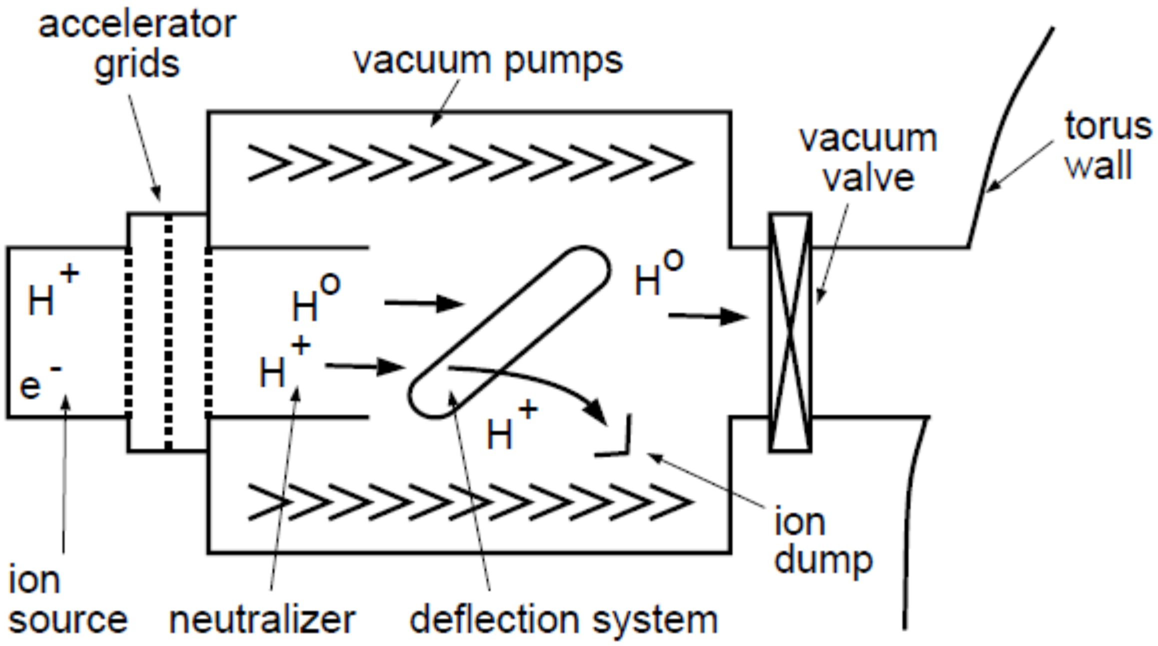
Neutral beam injection

Heating with neutral beam injection – process overview

Injection of a beam of neutral fuel atoms (H, D, T) with high energies (Eb > 50 keV)





TEXTOR NBI: up to 60 keV, 40 A neutral current, Pin = 2 MW
Co / counter beams

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Neutral beam injection

An energetic neutral particle beam is injected and is thus transferring energy into the plasma

- via ionization and subsequent thermalization / coulomb collisions:
  - $H_{Beam} + e \rightarrow H^{+} + e + e$ (electron collisions)
  - $H_{Beam} + p \rightarrow H^{+} + e + p$ (ion collisions)
- via charge exchange and and subsequent thermalization / coulomb collisions :
  - H_{Beam} (fast) + p_{Plasma} (slow) $\rightarrow$ H^{+} (fast) + H (slow)

All three reactions play a role when considering the penetration depth into the plasma:

$$\frac{dI_{Beam}}{dx} = -n\left(\sigma_{cx} + \sigma_{ion,i} + \frac{\langle \sigma_{ion,e} v_e \rangle}{v_{Beam}}\right) I_{Beam}$$

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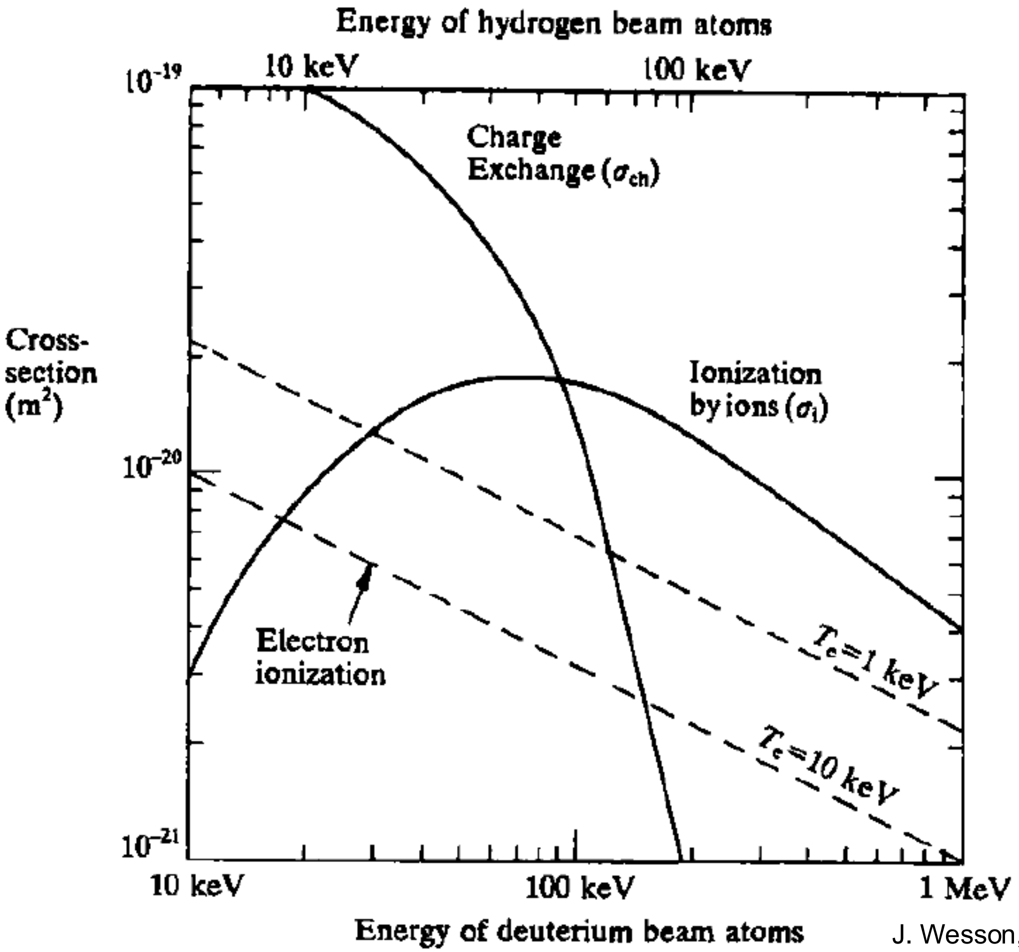
Neutral beam injection

Penetration depth

All three reactions play a role when considering the penetration depth into the plasma:

$$\frac{I_{Beam}(x)}{I_{Beam,0}} = e^{-n\left(\sigma_{cx}+\sigma_{ion,i}+\frac{\langle\sigma_{ion,e}v_e\rangle}{v_{Beam}}\right)x}$$



J. Wesson, Tokamaks

<u>Beam attenuation also caused due to</u>

- elastic collisions with plasma species
- collissions with impurities

→ Further decrease of penetration depth

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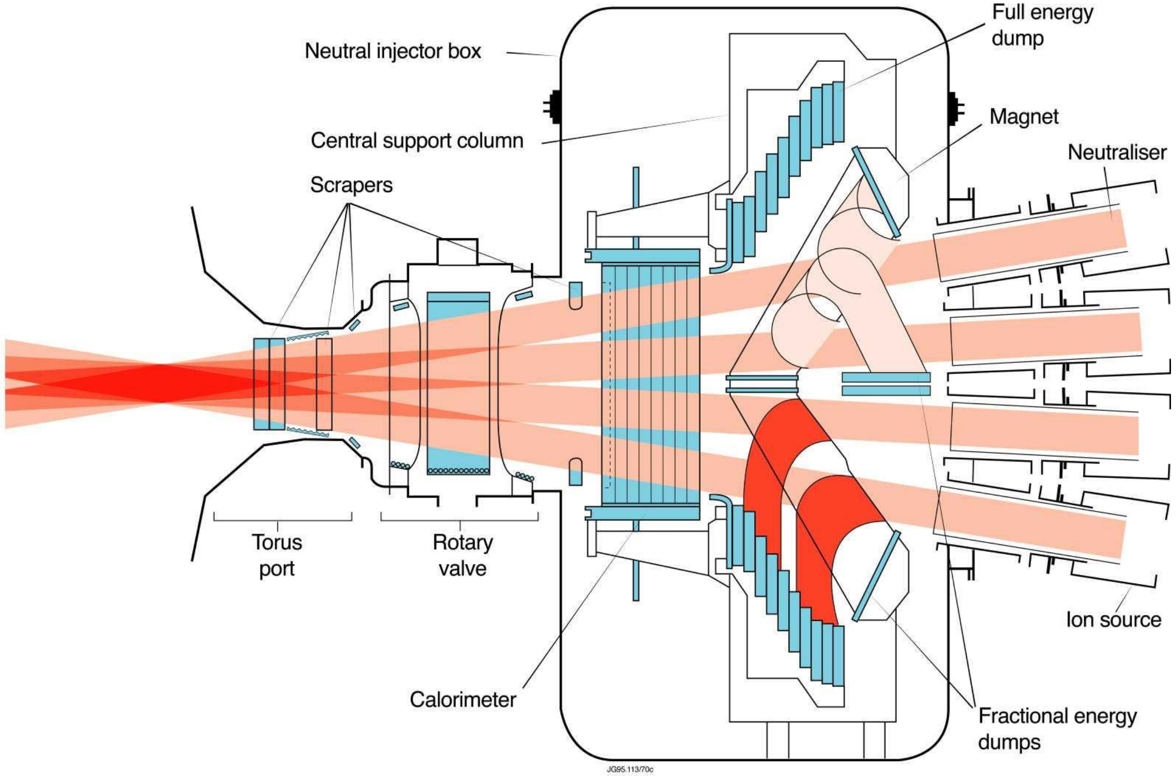
JET neutral beam system & components



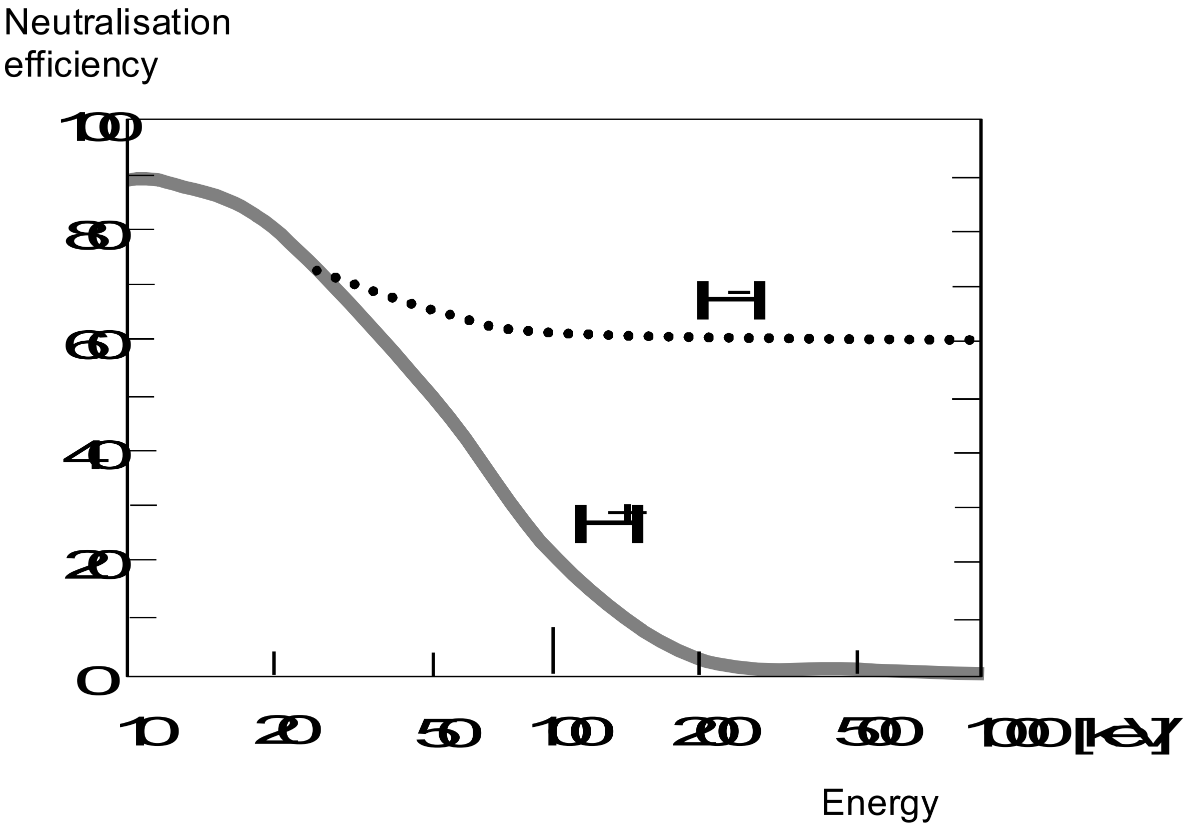
JET NB Injector cryopump

JET NB deflection magnet, residual ion dumps and calorimeter

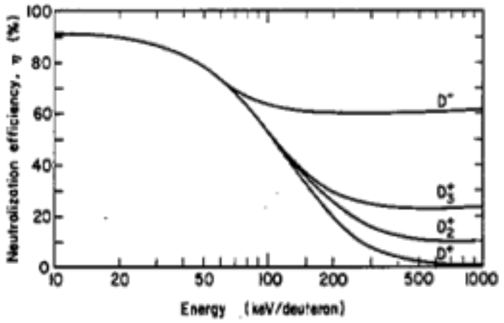
Neutral Beam Injection

Particle Energies and Physical Limits

- Problem for reactor NBI
  - Higher particle energies required (1 MeV for ITER)
  - neutralization efficiencies drop for higher energies
- Solution: Negative Ions



Efficiency:



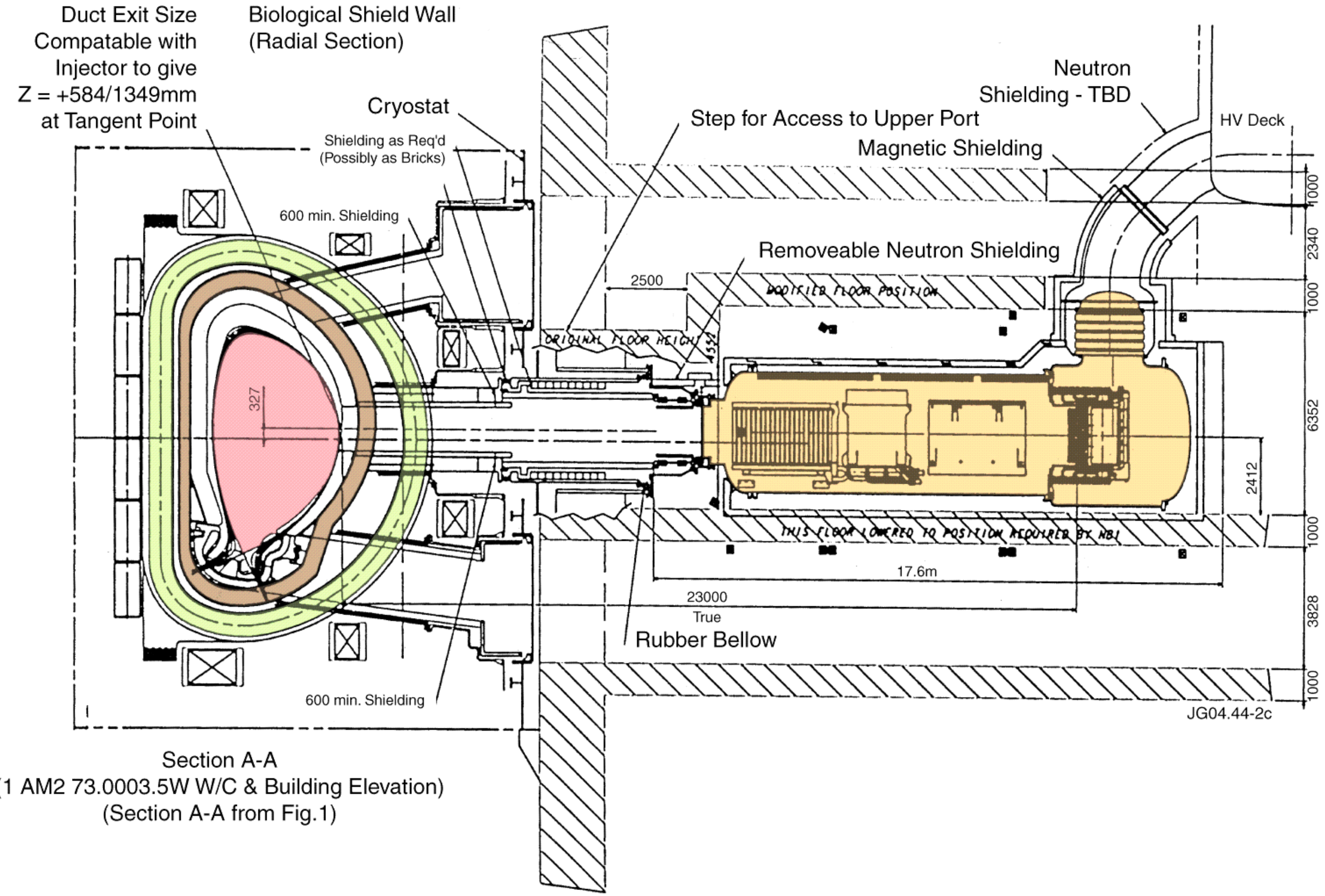
based on charge exchange

$$\underbrace{D^+}_{\text{fast}} + \underbrace{D_2}_{\text{gas}} \rightarrow \underbrace{D}_{\text{fast}} + \underbrace{D_2^+}_{\text{slow}}$$

balance against re-ionization

$$\underbrace{D}_{\text{fast}} + \underbrace{D_2}_{\text{gas}} \rightarrow \underbrace{D^+}_{\text{fast}} + \underbrace{D_2}_{\text{gas}} + \underbrace{e^-}_{}$$

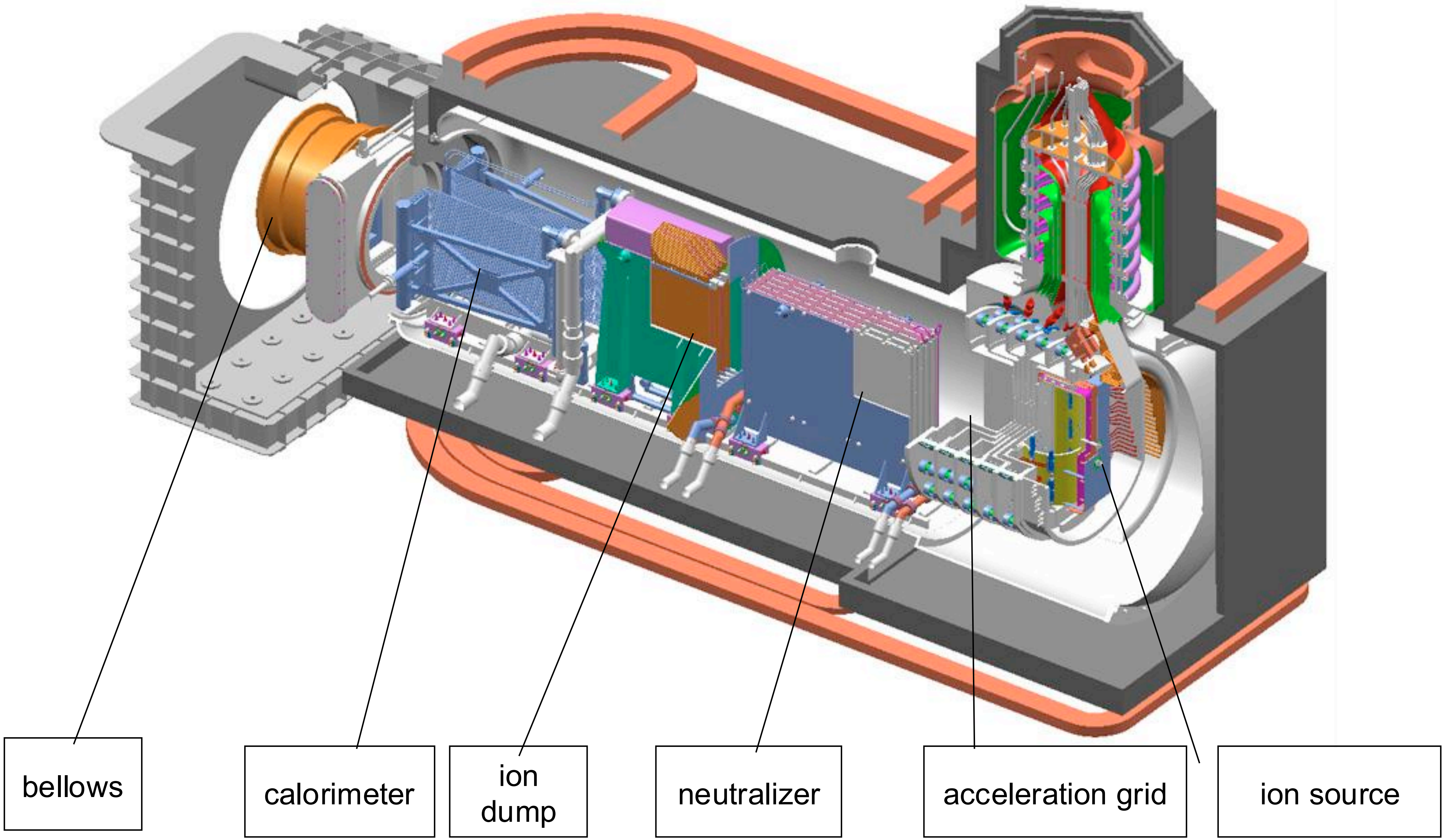
ITER NBI beamline



Section A-A
(1 AM2 73.0003.5W W/C & Building Elevation)
(Section A-A from Fig.1)

Neutral particle injector for ITER

E = 1 MeV
P = 2 x 16.5 MW



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Microwave heating

An aside: wave propagation and absorption

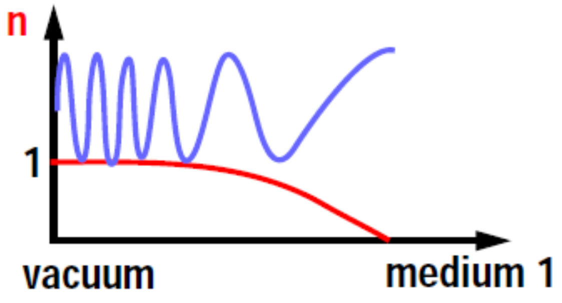
Dispersion relations describe how a wave moves through, responds to a medium

cutoff:

$n \rightarrow 0$

$v_{ph} \rightarrow \infty$

reflection



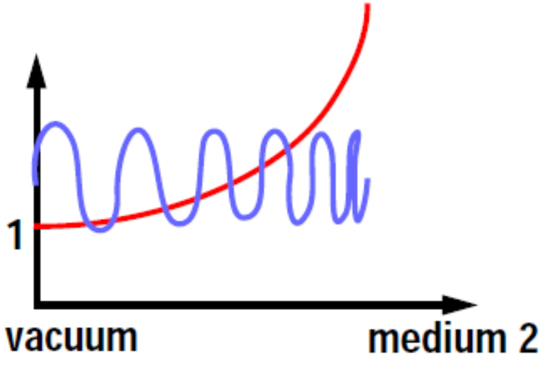
resonance:

$n \rightarrow \infty$

$v_{ph} \rightarrow 0$

singularity,

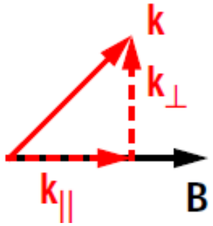
mode-conversion



dispersion relation for plane waves: $\underline{k} = \underline{k}(\omega)$

typically $k_{||}$ given, then $k_{\perp} = k_{\perp}(\omega, k_{||})$

index of refraction: $n = \frac{c}{v_{ph}} = \frac{ck}{\omega}$



Phase velocity:	$v_{ph}=\omega/k$
Group velocity:	$v_{Gr}= d\omega/dk$
Refractive index:	$n = c\, k / \omega$

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Overview of EM wave heating: many choices

ECRH:
electron cyclotron resonance heating

$$\omega_{ce} = \frac{eB}{m_e}$$

2 – 200 GHz

LH:
lower hybrid

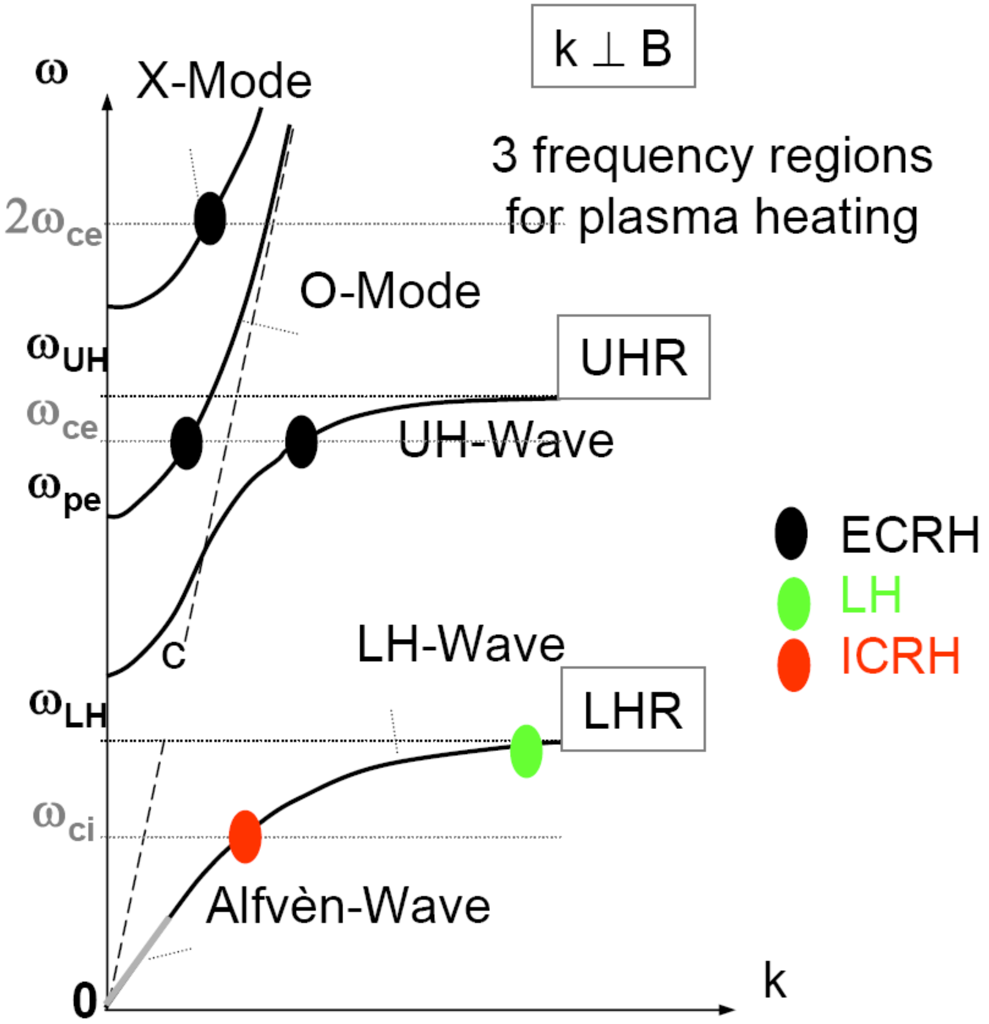
$$\omega_{LH} = \sqrt{\omega_{ce}\,\omega_{ci}}$$

2 – 5 GHz

ICRH:
ion cyclotron resonance heating

$$\omega_{ci} = \frac{eB}{m_i}$$

30 – 100 MHz



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Interaction of waves and particles: wave absorption

Collisionless damping allows energy exchange between plasma and electromagnetic waves

ν: collision frequency

$\omega \leq \nu$: low temperature plasmas

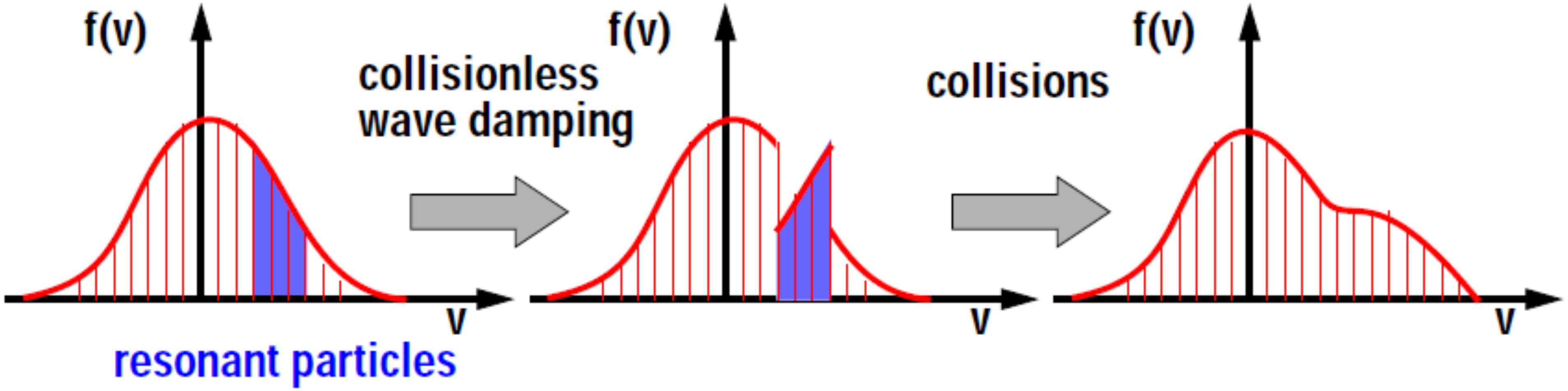
collisional damping

all particles involved

$\omega \geq \nu$: fusion plasmas

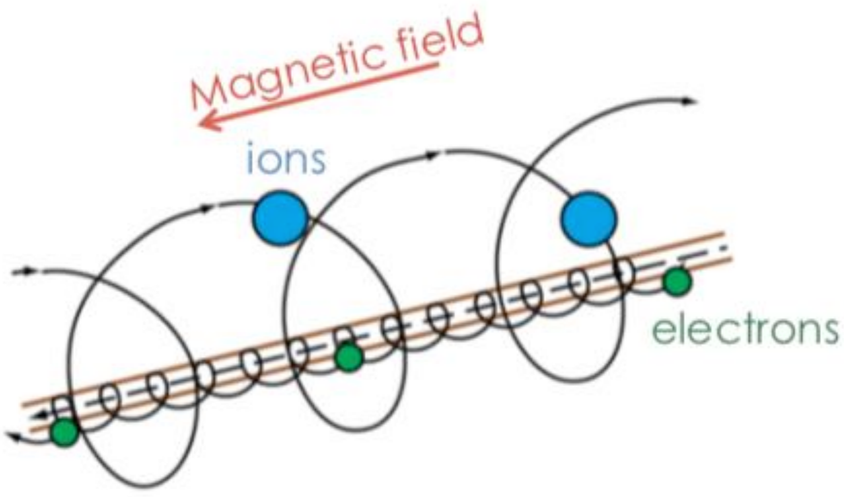
collisionless damping

only some resonant particles (electrons, ions) involved



Electron cyclotron range waves provide heating, drive current

- Electron cyclotron (EC) frequency is the natural frequency of rotation of electrons in magnetic fields $f_{ce} = \frac{eB}{2\pi m_e c}$

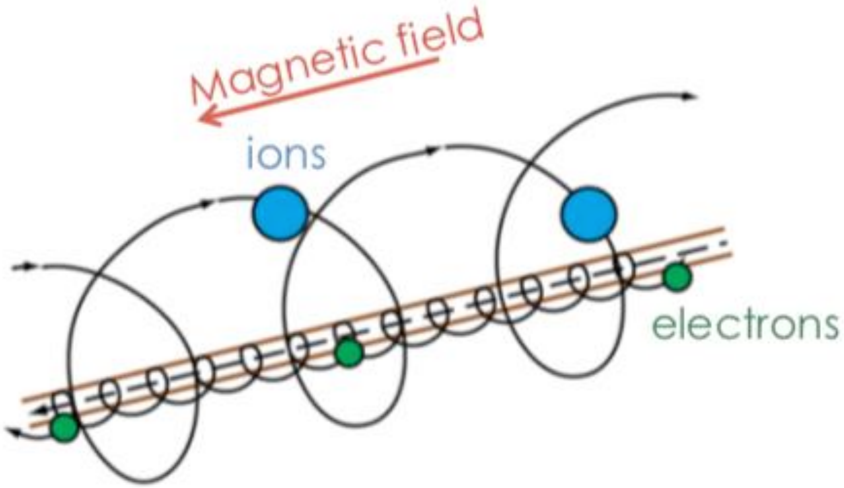


$$F = q(E + v \times B)$$

CYCLOTRON MOTION

Electron cyclotron range waves provide heating, drive current

- Electron cyclotron (EC) frequency is the natural frequency of rotation of electrons in magnetic fields $f_{ce} = \frac{eB}{2\pi m_e c}$
- Considering this frequency range, there are two solutions to the cold plasma dispersion relation
- Ordinary mode (O-mode)
  - E is parallel to B
  - Independent of B
  - Depends on density
- Extraordinary mode (X-mode)
  - E is perpendicular to B
  - Depends on B, n_e



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CYCLOTRON MOTION

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- Ordinary mode (O-mode)
  - E is parallel to applied B
  - Independent of B
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  - E is perpendicular to applied B
  - Depends on B, n_e

Both O-mode and X-mode have resonances at the electron cyclotron frequency, cutoffs depend on plasma density

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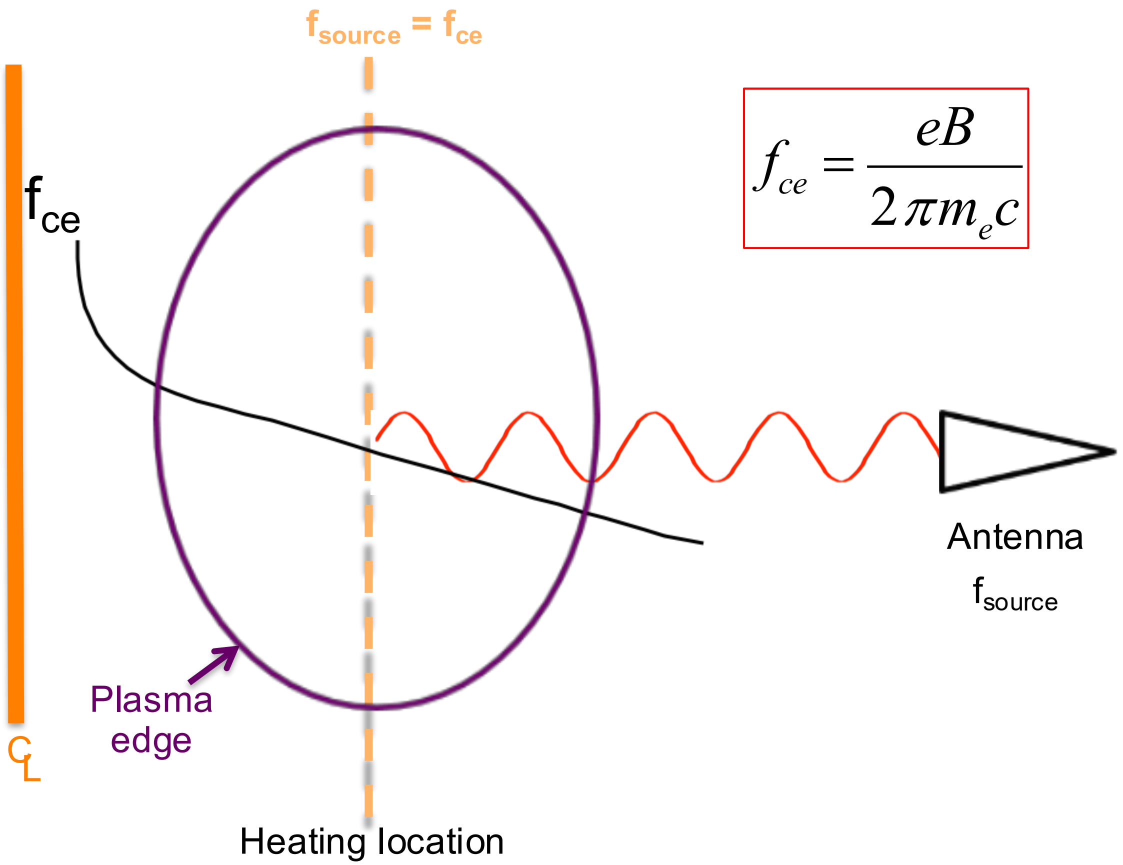
EC waves provide localized heating/current drive

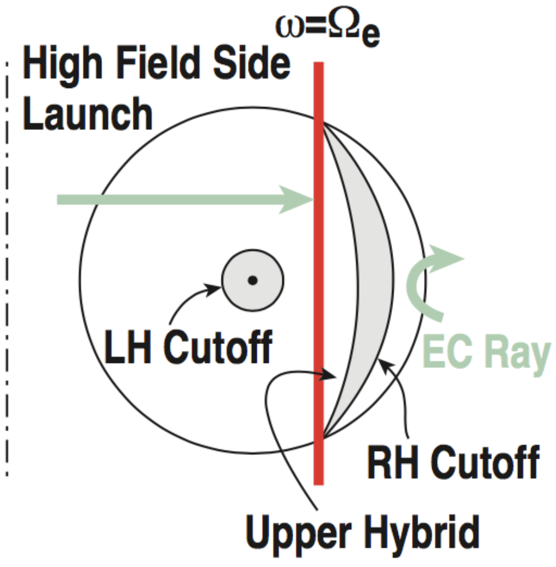
- Launched radiofrequency (RF) waves absorbed near cyclotron resonance
  - Tune to either electron or ion cyclotron motion
  - RF source frequency can be chosen to heat precise radius
  - For tokamaks, $B_t \propto 1/R$



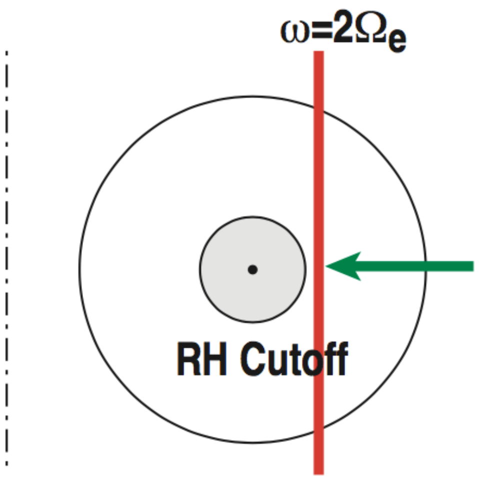
EC waves provide localized heating/current drive

- Can provide:
  - Electron heating
  - Current profile control, sustainment
  - Control of magnetohydrodynamic (MHD) activity

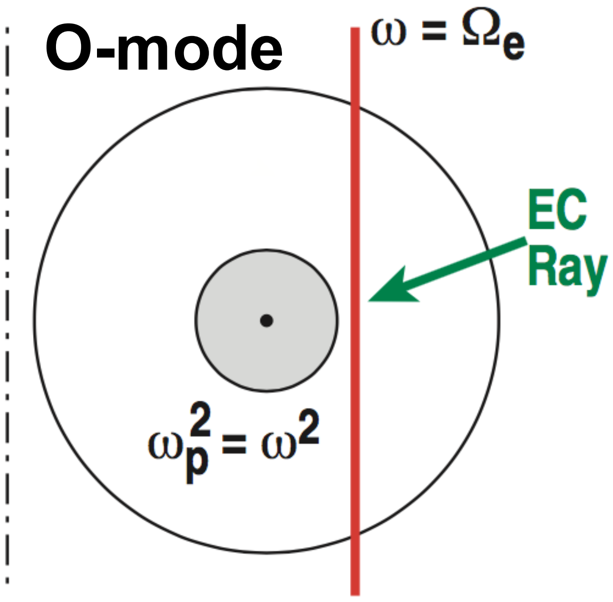
X-mode



Fundamental



Second Harmonic

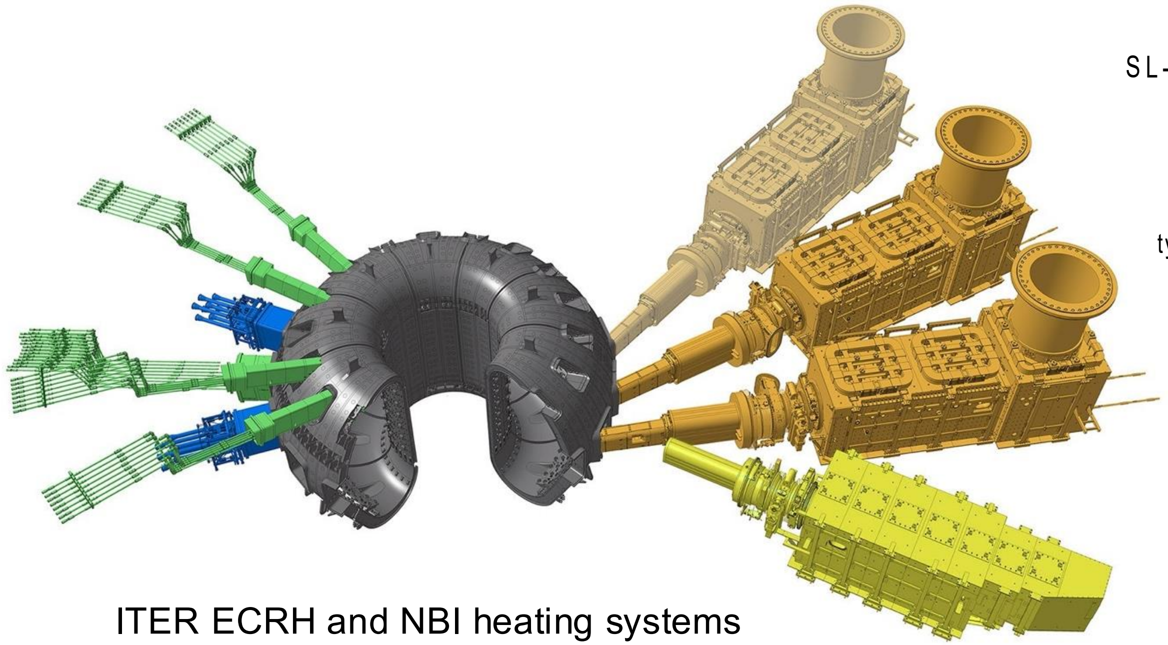


Fundamental

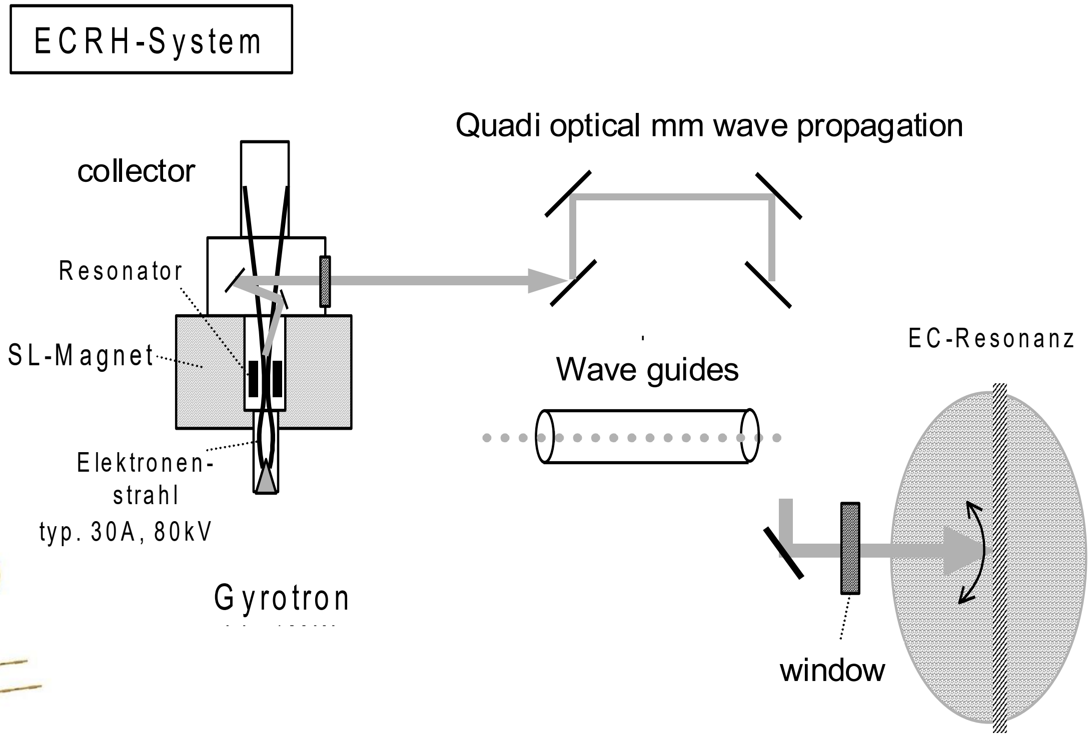
R. Prater PoP (2003)

Microwave systems can be located far from device

Couple to the plasma via waveguides or use quasi optical via mirrors using mm optics to bring waves into plasma



ITER ECRH and NBI heating systems



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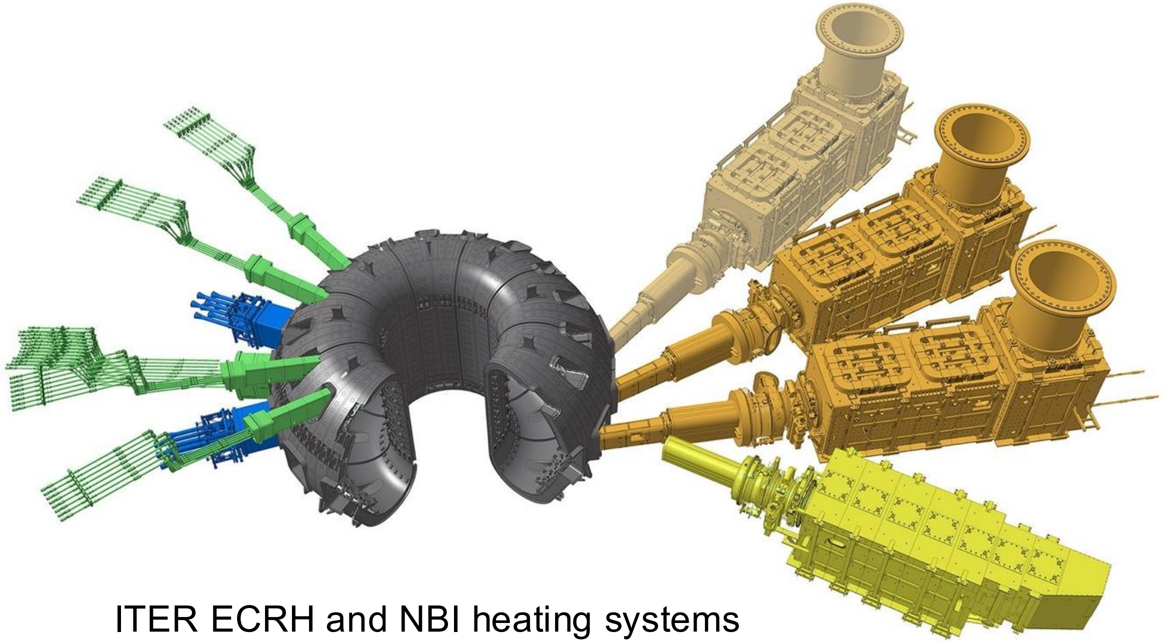

ITER ECRH and NBI heating systems

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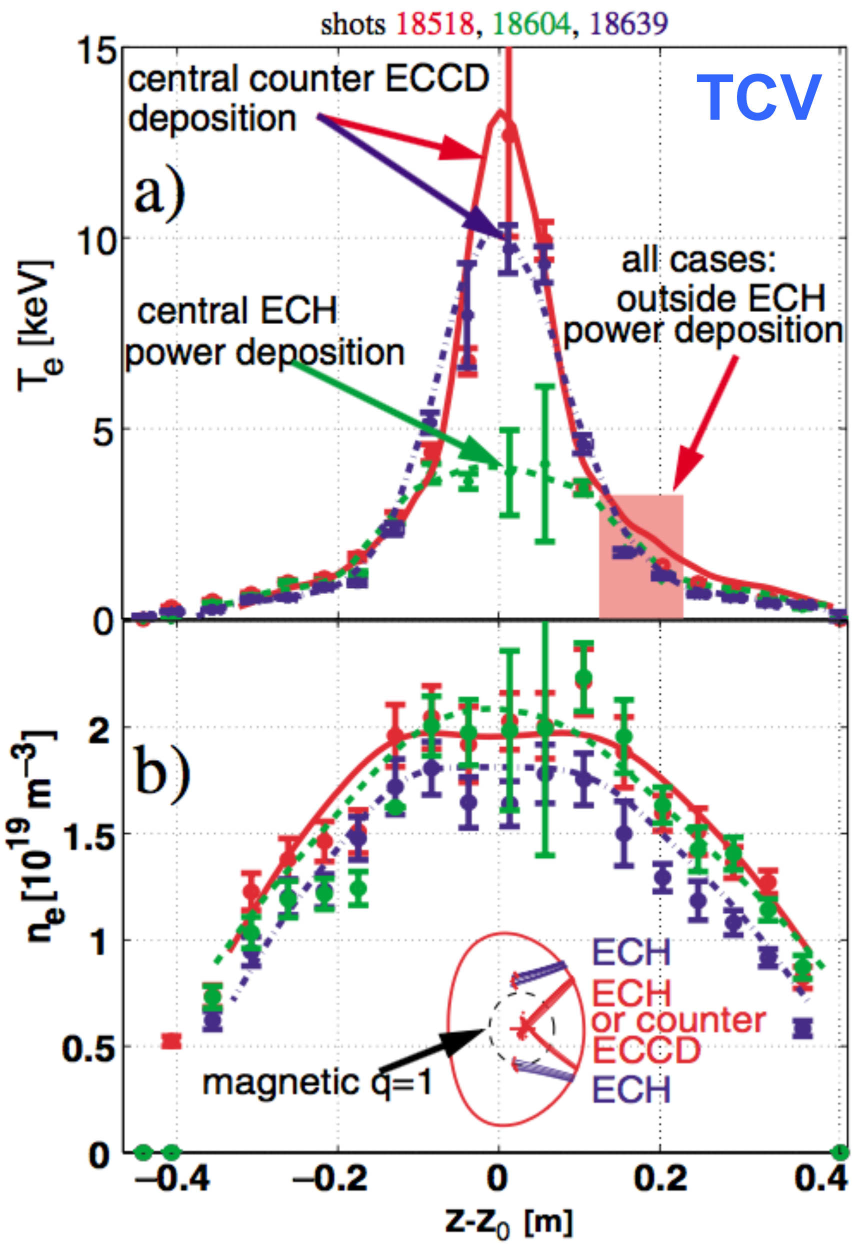
EC waves provide localized heating/current drive

- Many examples of ECH/ECCD in tokamaks and other confinement devices
  - Large-scale, high-performance devices depend on waves for heating

- EC heating/ EC current drive can provide current profile tailoring in TCV
  - Improve central electron energy confinement
  - Stabilize MHD modes



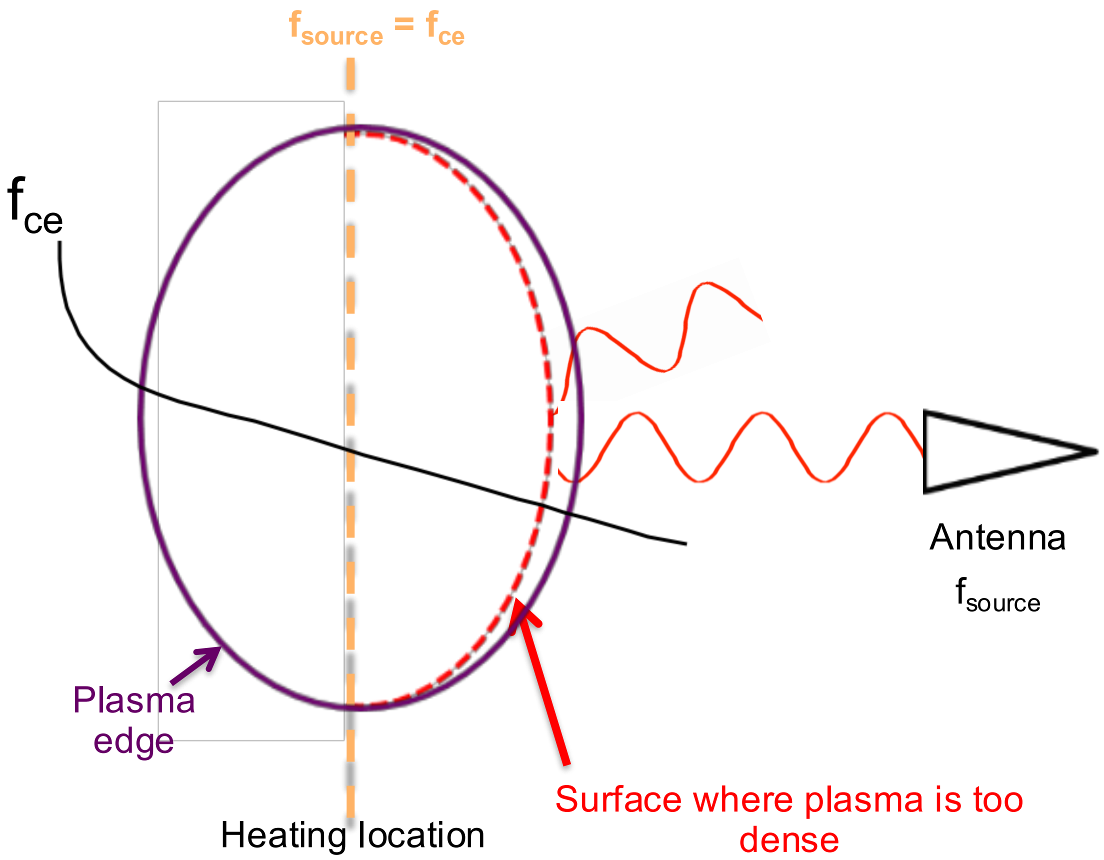
Z.A. Pietrzyk PRL 86, 8 (2001)

Electron cyclotron wave injection provides plasma heating, current drive – in certain conditions

- If plasma is too dense, O-mode and X-mode reflected near plasma edge
  - Happens in spherical tokamaks and stellarators

$$\omega_{source} > \omega_{pe}$$

- Alternative heating method required



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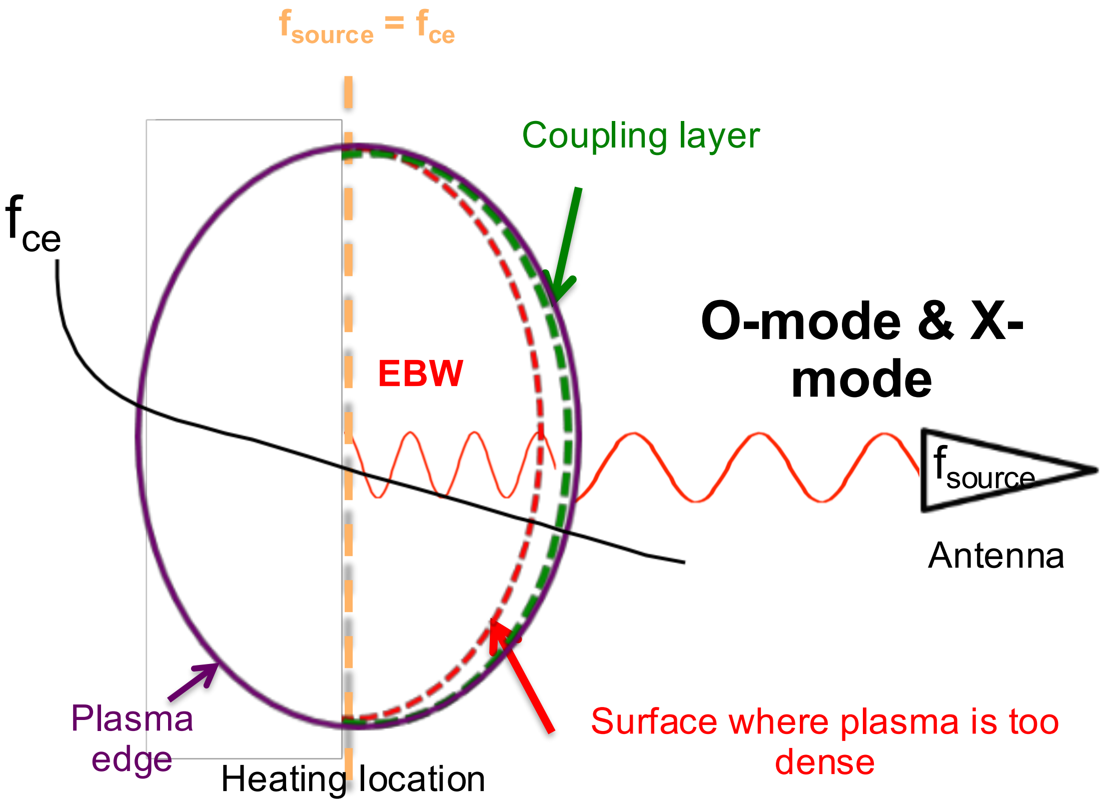
Electron Bernstein waves can travel in high density plasmas

- Electron Bernstein Waves (EBW) can only travel inside the plasma
  - Wave moves due to coherent motion of charged particles

- Can only couple to EBW by launching O- or X-modes



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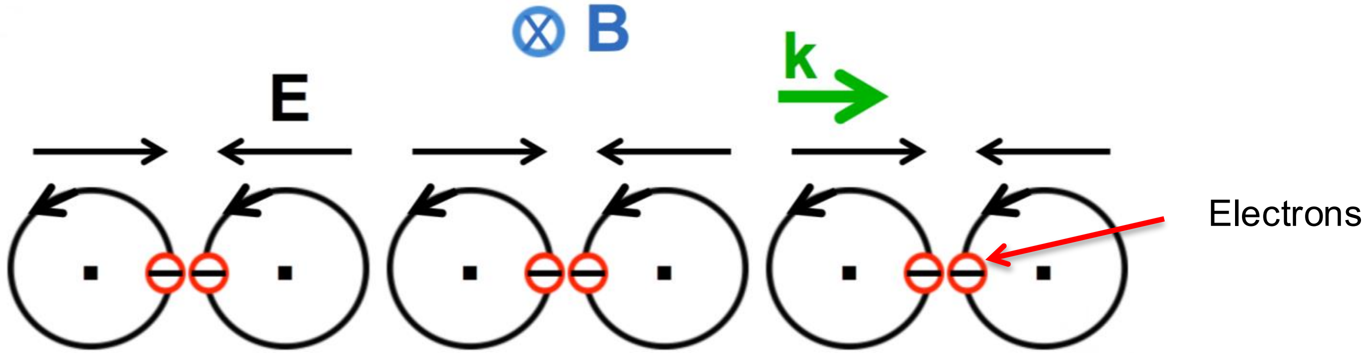
Electron Bernstein waves can propagate in overdense plasmas

- Electron Bernstein waves (EBW) are hot plasma waves:
  - Longitudinal, electrostatic waves
  - Propagates perpendicular to B
  - Do not experience a density cutoff in the plasma



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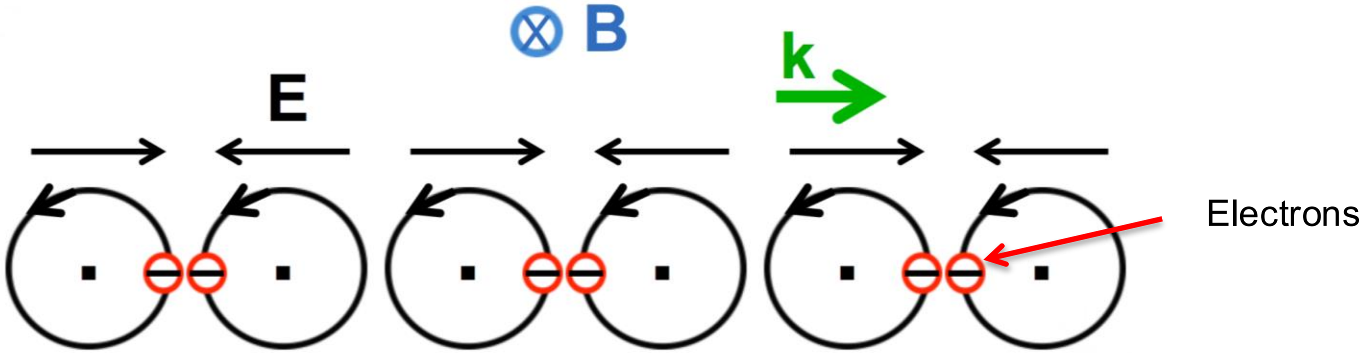
  - Propagates perpendicular to B
  - Do not experience a density cutoff in the plasma



- Cannot propagate in vacuum -> must launch O- or X-mode to mode couple to EBW

$$1-2\sum_s \frac{4\pi n_s m_s c^2}{\lambda B_0^2}\left[\sum_s e^{-\lambda} I_n(\lambda)\frac{n^2}{\left(\omega/\Omega\right)^2 - n^2}\right]=0 \quad \text{Where: } \lambda = \frac{k_\perp^2 \kappa T_\perp}{m\Omega^2}$$

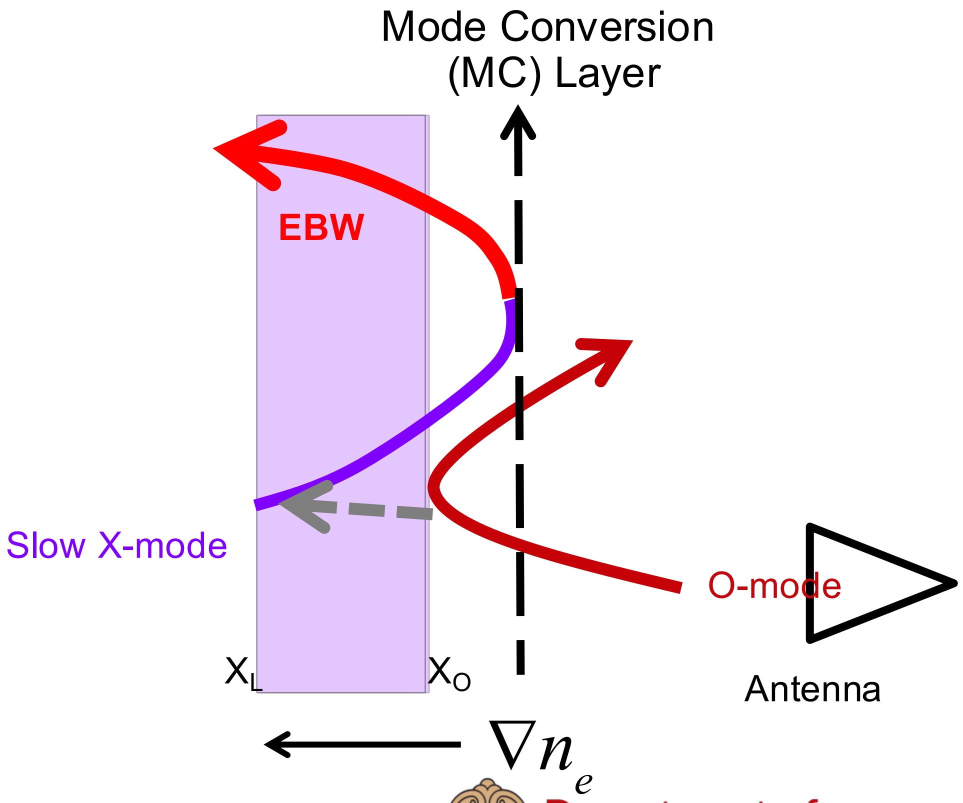
- As wave frequency approaches EC harmonic, $\omega = n\Omega_C$, wave is strongly absorbed

Electron Bernstein waves can propagate in overdense plasmas

- Electron Bernstein waves (EBW) are hot plasma waves:
  - Perpendicularly propagating, $k_{||} = 0$
  - Do not experience a density cutoff in the plasma
  - Longitudinal, electrostatic waves
  - Cannot propagate in vacuum
  - Absorbed near cyclotron harmonics

- EBW coupling efficiency depends on plasma parameters near plasma edge
  - Density gradient
  - Magnetic field pitch

O-X-B Coupling



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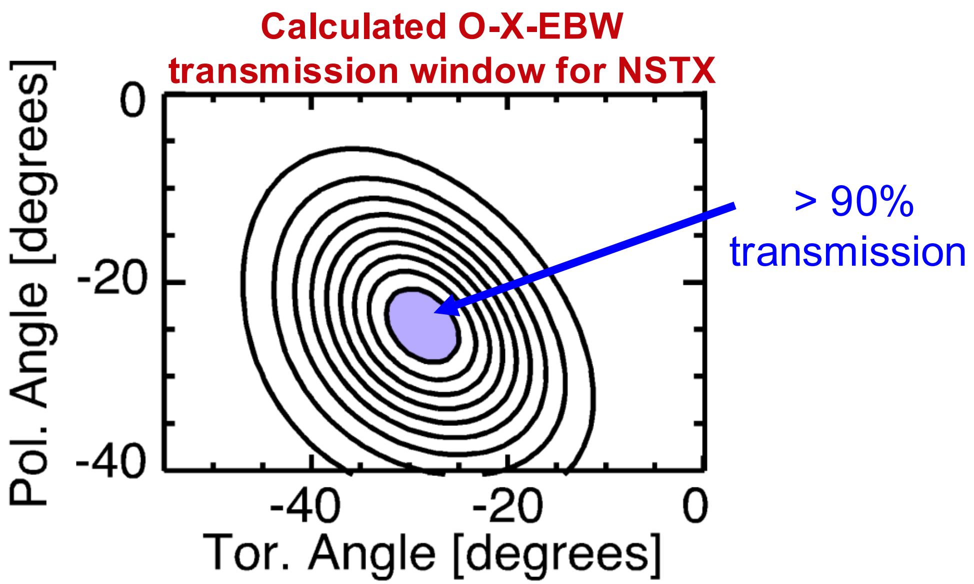
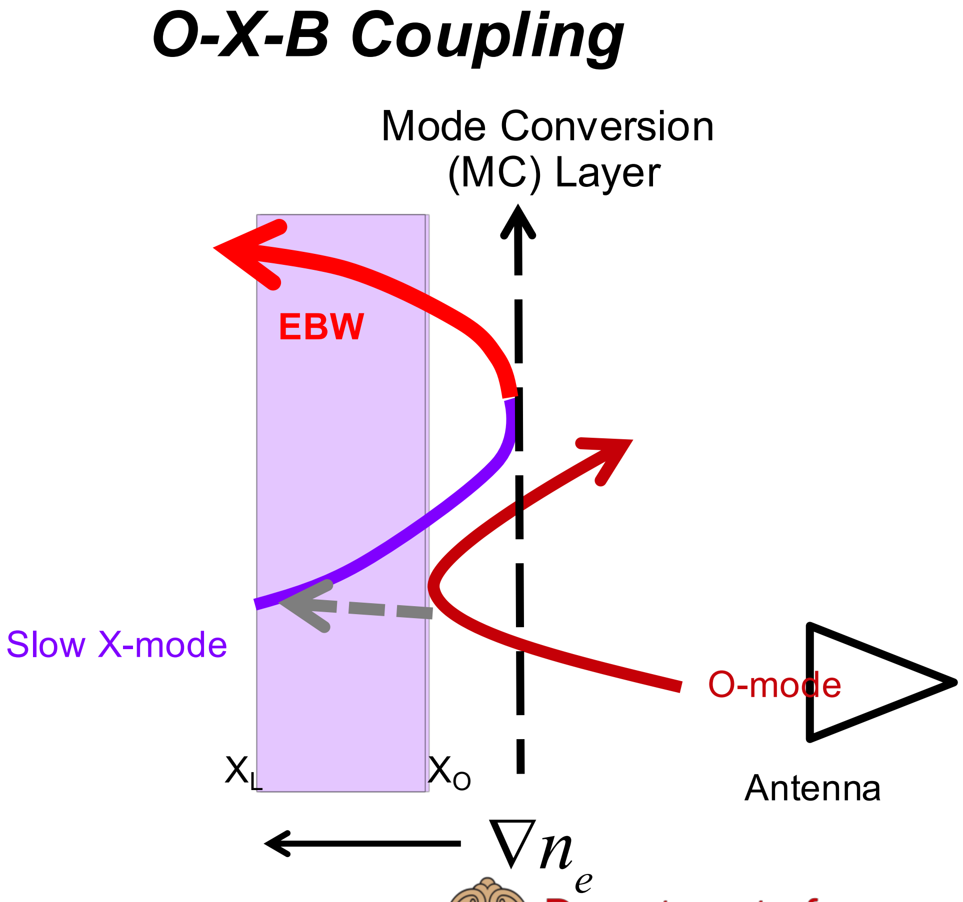
Electron Bernstein waves can propagate in overdense plasmas

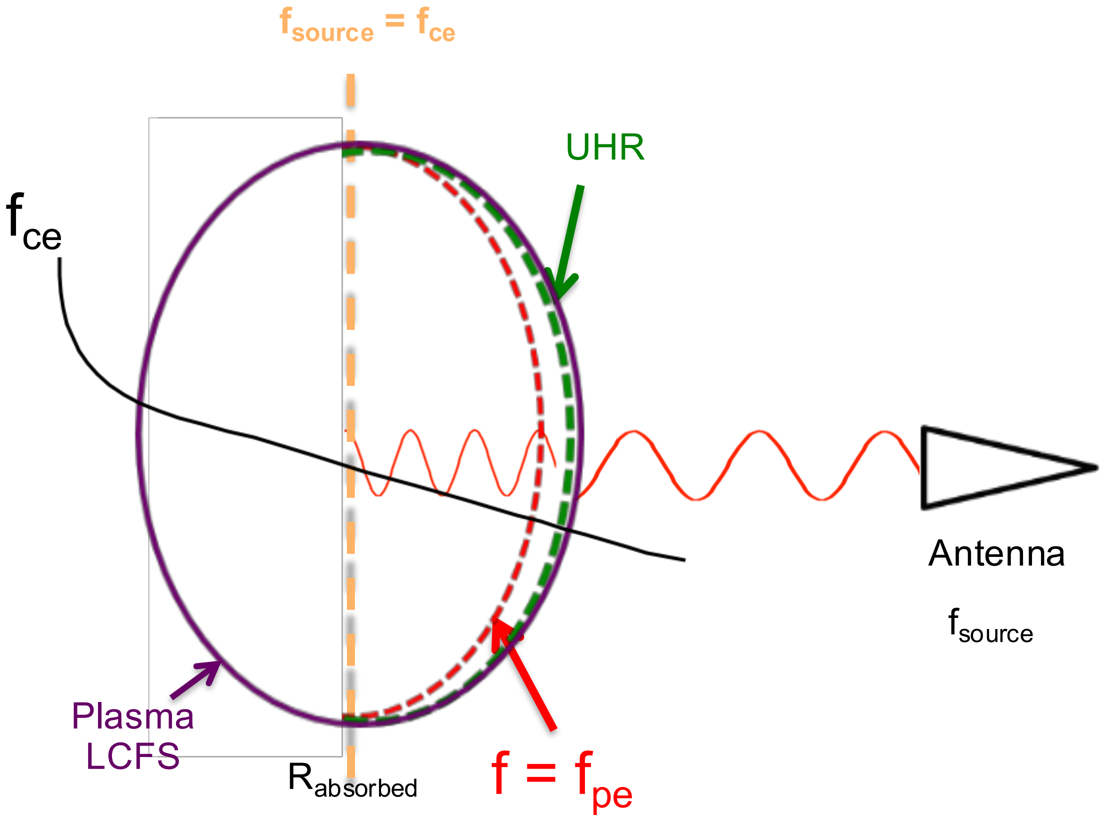




- EBW emission data can be used to provide coupling efficiency and polarization information to design heating, current drive system

EBW emission can be used to measure temperature

Electron Bernstein wave emission at blackbody levels, proportional to local T_e



Larmor formula: $$I_\omega = \frac{\omega^2 k_B T_{rad}}{8\pi^3 c^2}$$

Measured T_{rad} is proportional to local T_e

$f_{ce} \sim 1/R$ ➔ radial localization

Diem et al, PRL (2009)

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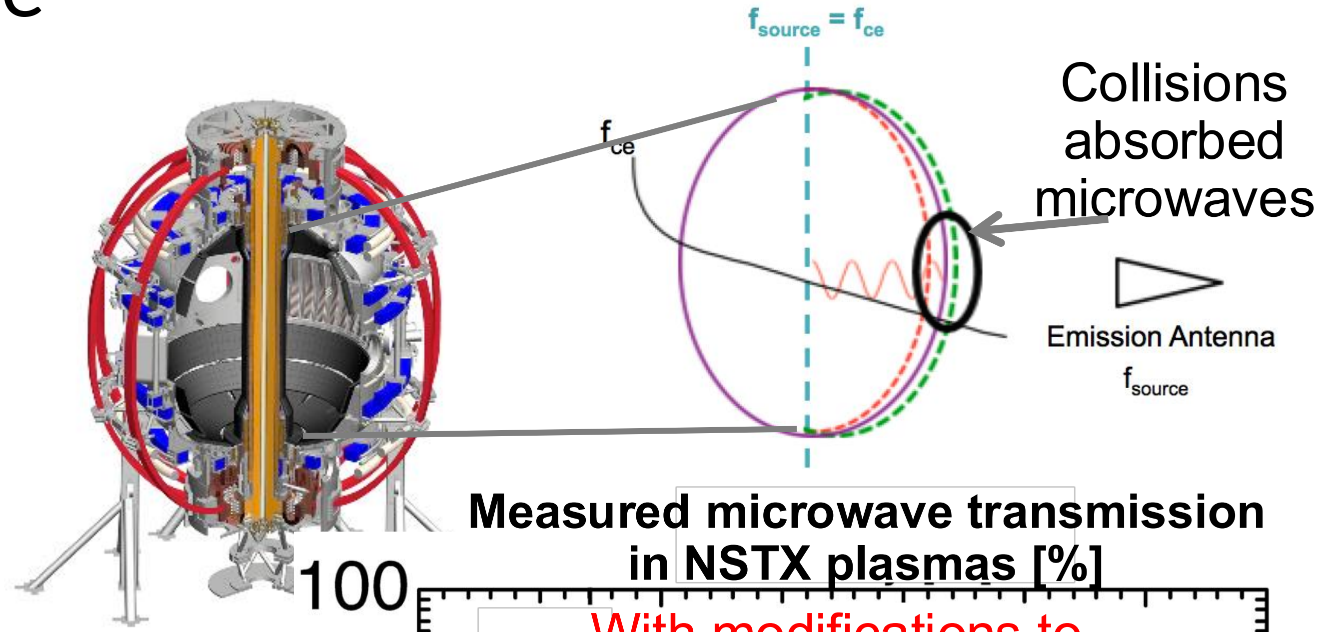
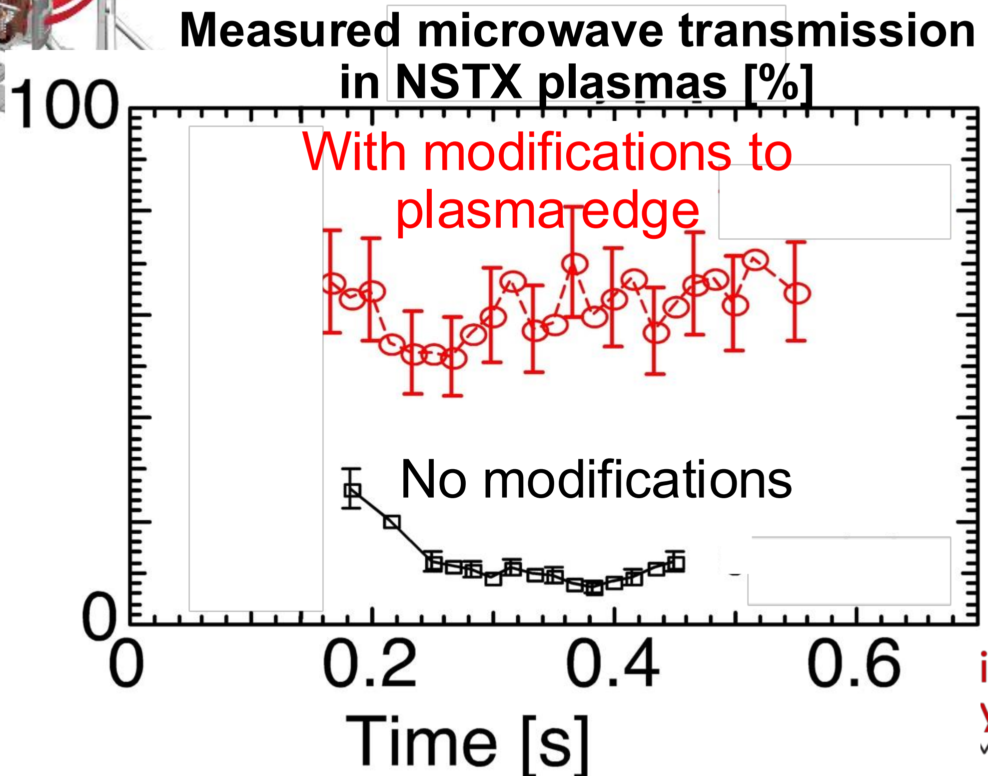
Coupling microwave power to high density fusion plasmas can be difficult - but possible

- Plasma naturally emits microwaves from cyclotron resonance location

- Assumed physics of microwave emission from high density plasmas same as launching
  - Measurements on NSTX didn't agree with predictions
  - Plasma edge had too many collisions, absorbed microwaves

- ***Unexpected results present opportunities***

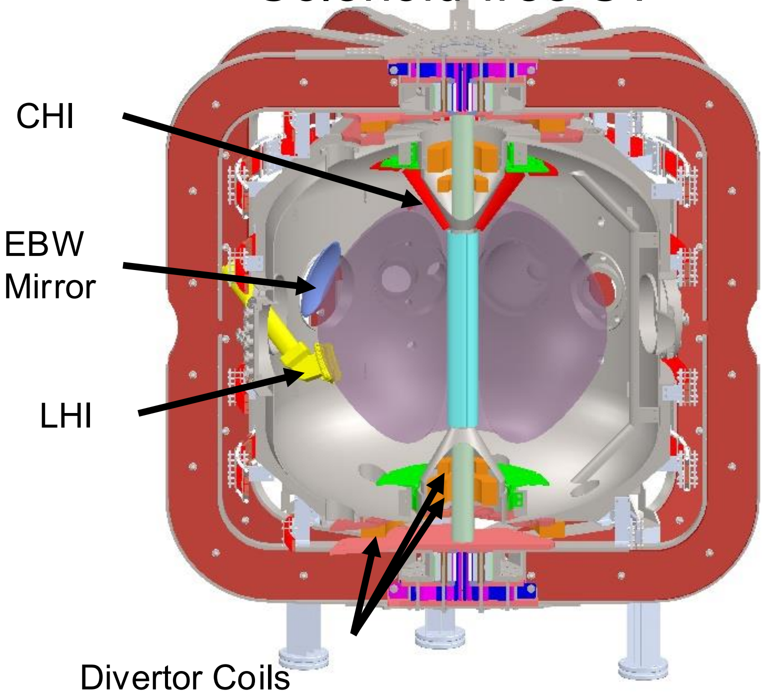




Pegasus-III – lighting a match for fusion

- Future spherical tokamaks call for solenoid-free operation
  - Need to minimize solenoid due to shielding/cost
- Solenoid removal simplifies tokamak design
  - Potential cost reduction
  - More space for inboard shielding/blanket
  - Lower electromechanical stresses
- Pegasus is focused on developing several solenoid-free heating and current drive techniques
  - Local helicity injection
  - Coaxial helicity injection
  - EBW startup and sustainment
- University-class fusion facilities provide innovative approaches to fusion energy development

PEGASUS-III
Solenoid-free ST



PEGASUS Website: Publications, Presentations
http://pegasus.ep.wisc.edu

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Overview of plasma heating

- **Ohmic heating**
  - **$P = V_{loop} * I_{plasma}$**
  - **Dissipative heating of current**
  - **Heats electrons**
- **Neutral beam injection (NBI)**
  - **Injection of high energetic neutral fuel atoms into plasma**
  - **Heating electrons and ions**
- **Electron cyclotron resonance heating**
  - **Inject microwaves with $\omega = \omega_{ce}$**
  - **Heats electrons**
- Lower hybrid resonance heating
  - Inject waves with $\omega = (\omega_{ce}\omega_{ci})(1/2)$
  - Heats electrons and ions
- Ion cyclotron resonance heating
  - Injection of radio frequency waves with $\omega = \omega_{ci}$ (10s of MHz)
  - Heats ions
- **DC helicity injection**

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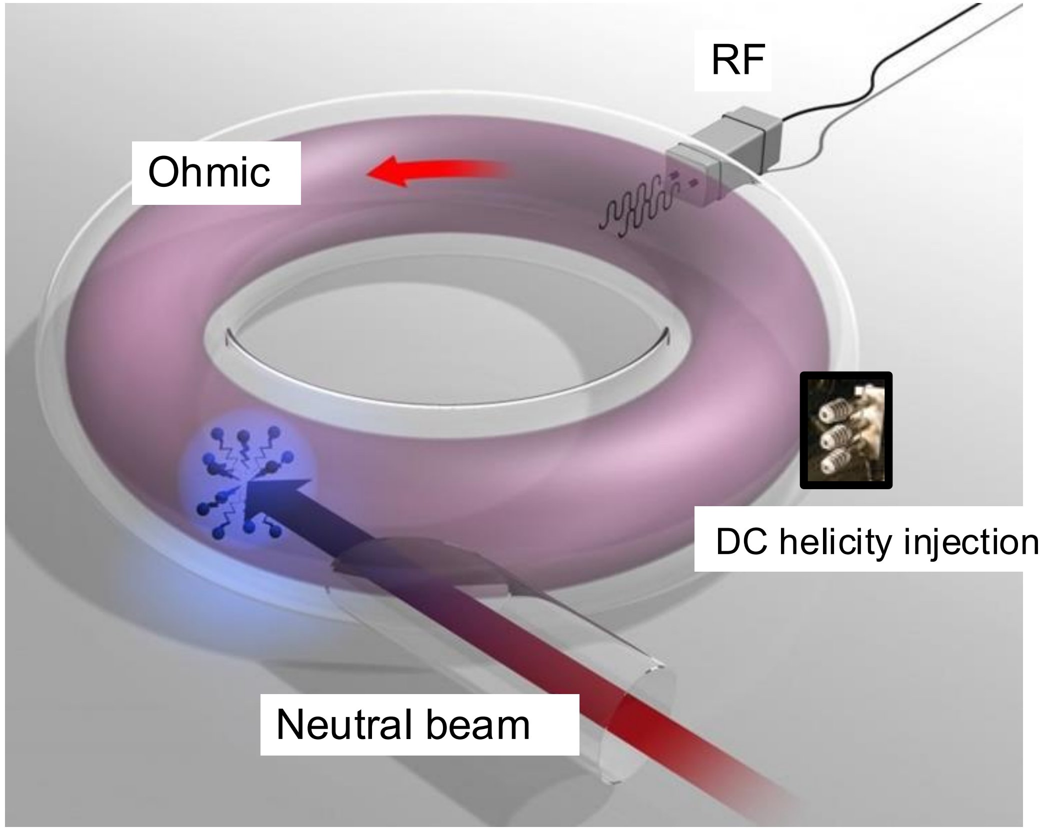
Tokamak plasmas require current drive and heating to achieve fusion

- Reaching ignition requires threshold for:

$$n \bullet T \bullet \tau_E$$

  - External heating required to reach temperature for ignition
  - After ignition, self-heating sustains plasma

- Several methods of external heating & current drive available
  - Electron cyclotron (EC) resonance is in microwave range of frequencies
  - At high densities, injected microwave can be reflected – requires alternative methods of coupling microwave power
  - **DC helicity injection**



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Why investigate startup for tokamaks?

- Compact toroidal geometry challenge for central solenoid
  - Also beneficial for ATs to minimize need for central solenoid
- Drives the search for non-solenoidal startup techniques
- Pegasus-III studies innovations in plasma startup techniques
  - Can we develop new startup methods to help reduce cost, complexity of future fusion power plants?
  - Are there any additional uses for this technology?

Pegasus-III Experiment

- New facility, 6.5 million Euros, for research in fusion energy and plasma science, UW-Madison
- Research team of 20+ people, funded at 2.5+ million Euros per year
- No ohmic solenoid!

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Helicity injection techniques can initiate and drive tokamak plasmas

$$K = \int_V \boldsymbol{B} \cdot \boldsymbol{A} \, dV$$

- Magnetic helicity, K ≡ "linkedness" of magnetic flux

- In a tokamak, K_{plasma} results from linking toroidal and poloidal fluxes
  - Total $K \propto I_p \Phi$
  - Injecting K → increases I_p

- Two methods of adding helicity:
  - **AC helicity injection** – increasing flux via magnetic induction within target volume
  - **DC helicity injection** – potential applied along open field lines that penetrate magnetic boundary

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Helicity balance and magnetic relaxation determine $I_p(t)$

- Helicity Balance:

$$\frac{dK}{dt} = -2\Phi\frac{\partial\Psi}{\partial t} - 2\oint_S dS(\varphi\vec{B}\cdot\hat{n}) - 2\int_V dV(\vec{E}\cdot\vec{B})$$

AC helicity injection rate

DC helicity injection rate

Helicity dissipation rate

$$\sim 2\Phi V_{IND} \quad + \quad \sim 2\Phi V_{LHI} \quad > \quad \sim 2\int_V dV(\eta\vec{J}\cdot\vec{B})$$

Helicity balance and magnetic relaxation determine $I_p(t)$

- Helicity Balance:

$$\frac{dK}{dt} = -2\Phi\frac{\partial\Psi}{\partial t} + 2\oint_S dS(\varphi\vec{B}\cdot\hat{n}) - 2\int_V dV(\vec{E}\cdot\vec{B})$$

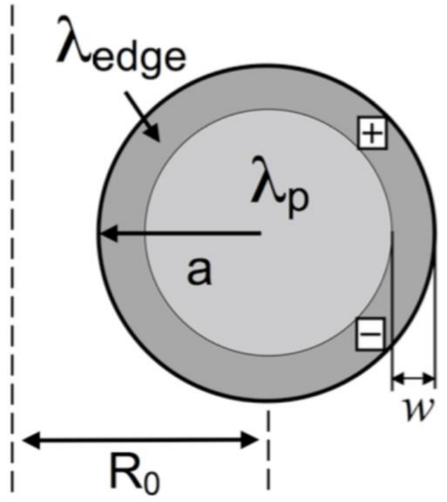
AC helicity injection rate | DC helicity injection rate | Helicity dissipation rate

$$\sim 2\Phi V_{IND} \quad + \quad \sim 2\Phi V_{LHI} \quad > \quad \sim 2\int_V dV(\eta\vec{J}\cdot\vec{B})$$

- Magnetic relaxation drives system to a minimum energy state while conserving K
  - Taylor/Relaxed state: $\nabla\times\vec{B} = \lambda\vec{B}$, where λ is constant throughout plasma

$$I_p \le I_{TL} \approx f_g\left[\frac{\varepsilon A_p I_{inj} I_{TF}}{2\pi R_{edge} w_{inj}}\right]^{1/2}$$

Simplified plasma geometry for Taylor limit approximation



$$\lambda(r) \equiv \frac{\mu_0\vec{J}\cdot\vec{B}}{B^2}$$

LHI Supplies Magnetic Helicity by Direct Current Injection

- Magnetic helicity K injected into the system[1]: $K = \int \boldsymbol{A} \cdot \boldsymbol{B} \; dV$

- Helicity balance: $\frac{dK}{dt} = -\,2\psi_T \frac{\partial \psi_p}{\partial t} - 2 \oint_S \phi \boldsymbol{B} \cdot \hat{n} \, dS - 2 \int_V \eta \, \boldsymbol{J} \cdot \boldsymbol{B} \; dV$

AC Helicity Injection | DC Helicity Injection | Helicity Dissipation

- Injecting current increases poloidal flux ψ_p linked with toroidal flux ψ_T from external coils
- Injection must be higher than dissipation for current drive

[1]Berger, Plasma Phys. Control. Fusion **41** B167 (1999)

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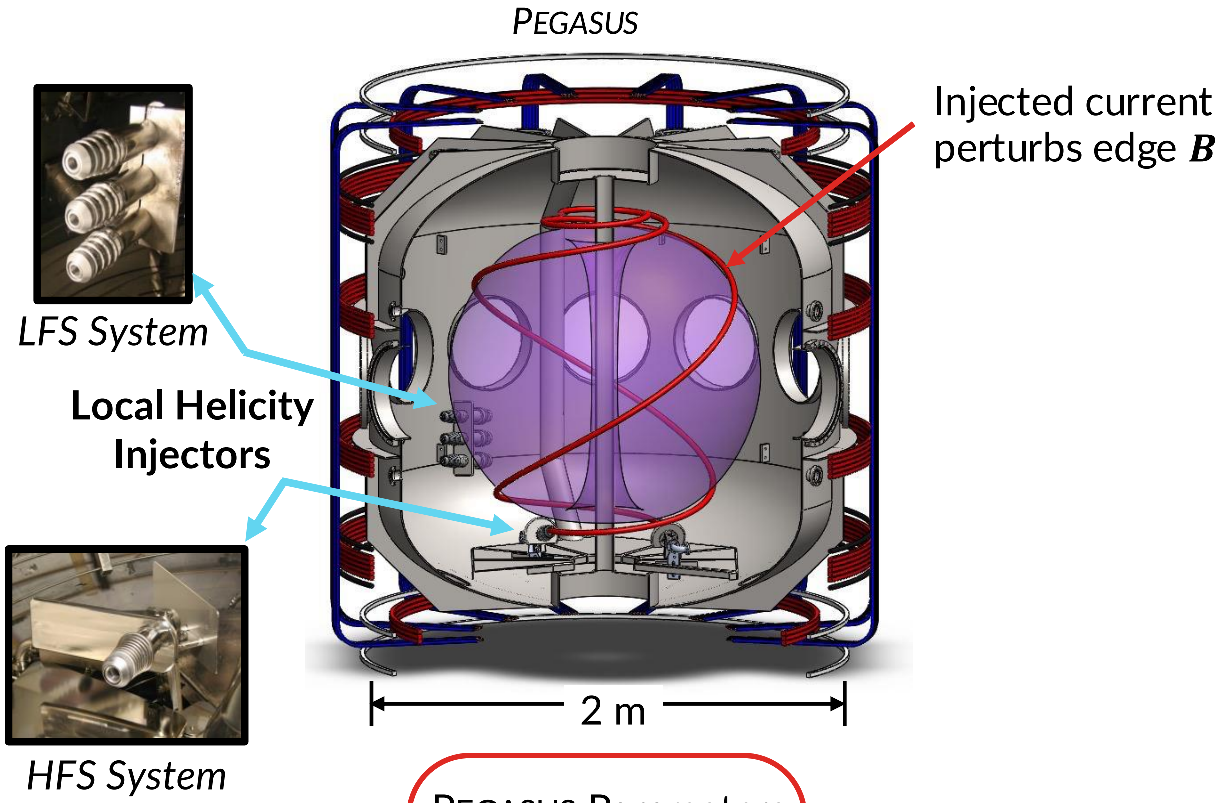
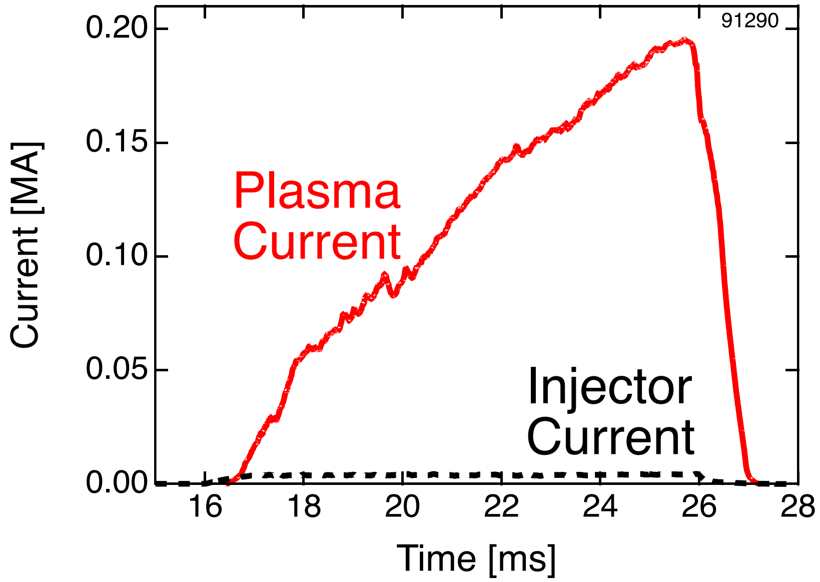
Local Helicity Injection (LHI) Provides Nonsolenoidal Startup



PEGASUS Parameters	
A	1.15-1.3
R_0 [m]	0.2—0.45
a [m]	≤ 0.4
κ	1.4—3.7

High Current Multiplication: $I_p \leq 0.2$ MA with $I_{inj} \leq 8$ kA



- Edge current extracted from small, modular injectors
- Unstable current streams relax to tokamak-like state via helicity conserving instabilities

M.W. Bongard, et al, Nuclear Fusion (2019), J.M. Perry, et al, Nuclear Fusion (2018)

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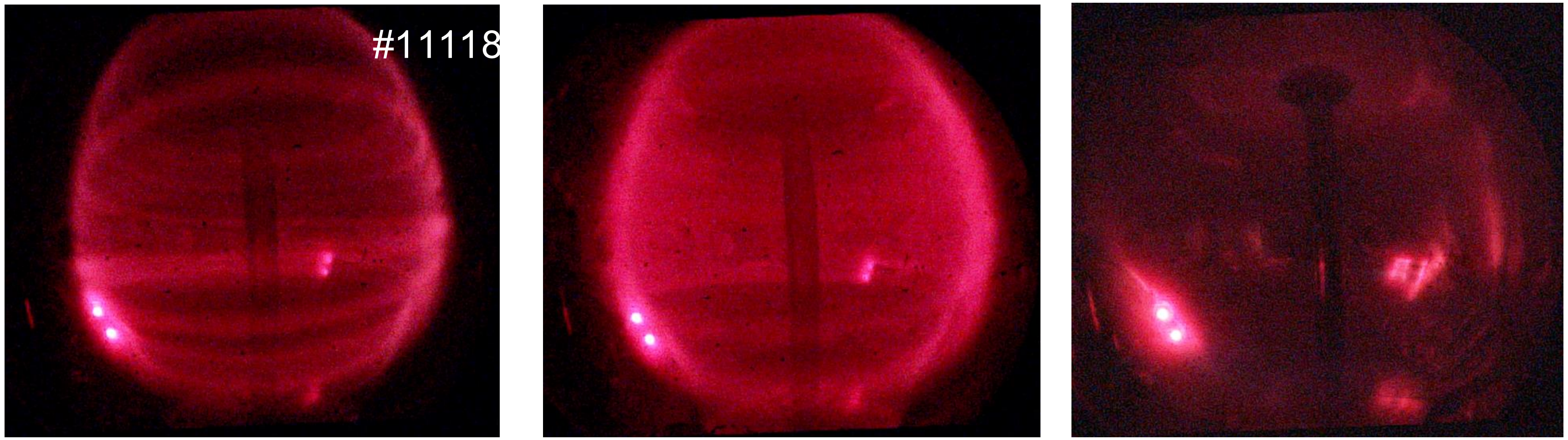
LHI initiates a tokamak-like system

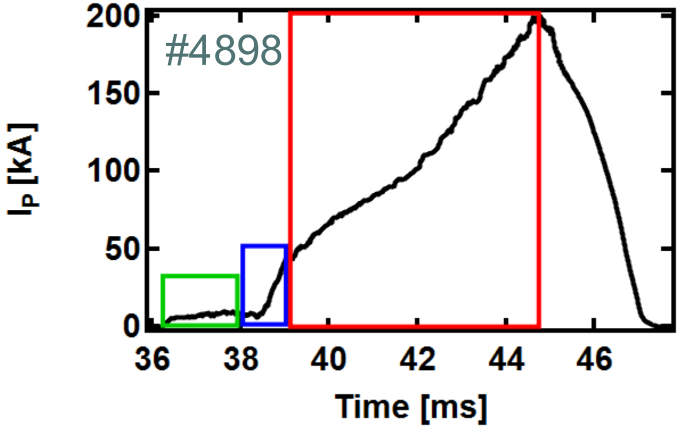
Fast Camera Images of LHI Stages on Pegasus-III



$I_p \approx N_{turns} I_{inj}$	$I_p \gtrsim N_{turns} I_{inj}$	$I_p \gg N_{turns} I_{inj}$
1. Injected current follows helical $\vec{B}$	2. Current streams go unstable, reconnect	3. Plasma relaxes to a “tokamak-like” state

Majority of magnetic energy is in axisymmetric mode

LHI-c I_P Trace



Typical parameters:

- $n_{e0} \sim 2 \times 10^{19}\ m^{-3}$
- $T_e \sim 30 - 100\ eV$

Image processing by T. Tierney

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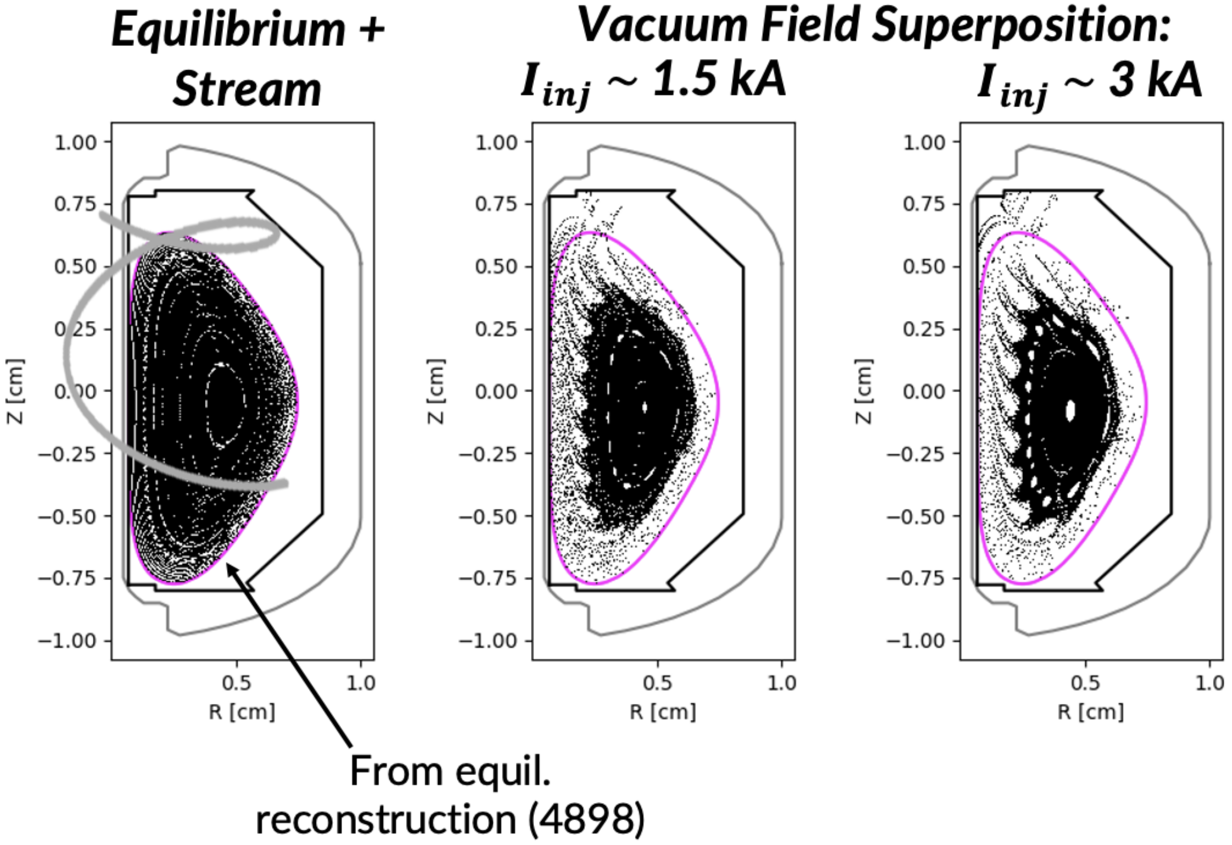
LHI open-field currents affect B-field structure

- Goal to understand/develop:
  - Plasma response to LHI
  - Information to develop predictive model of LHI plasma evolution
  - Feasibility of LHI for ELM mitigation, etc.

- Simple model of I_{inj} on helical vacuum field gives basic perturbation

- Measured toroidal variation of perturbed field confirms basic perturbation structure

- Basic field line following (no plasma response) shows significant effects on closed flux surface region



$$\boldsymbol{B}_{total} = \boldsymbol{B}_{equil} + \boldsymbol{B}_{pert,\,vacuum} + \boldsymbol{B}_{response}$$

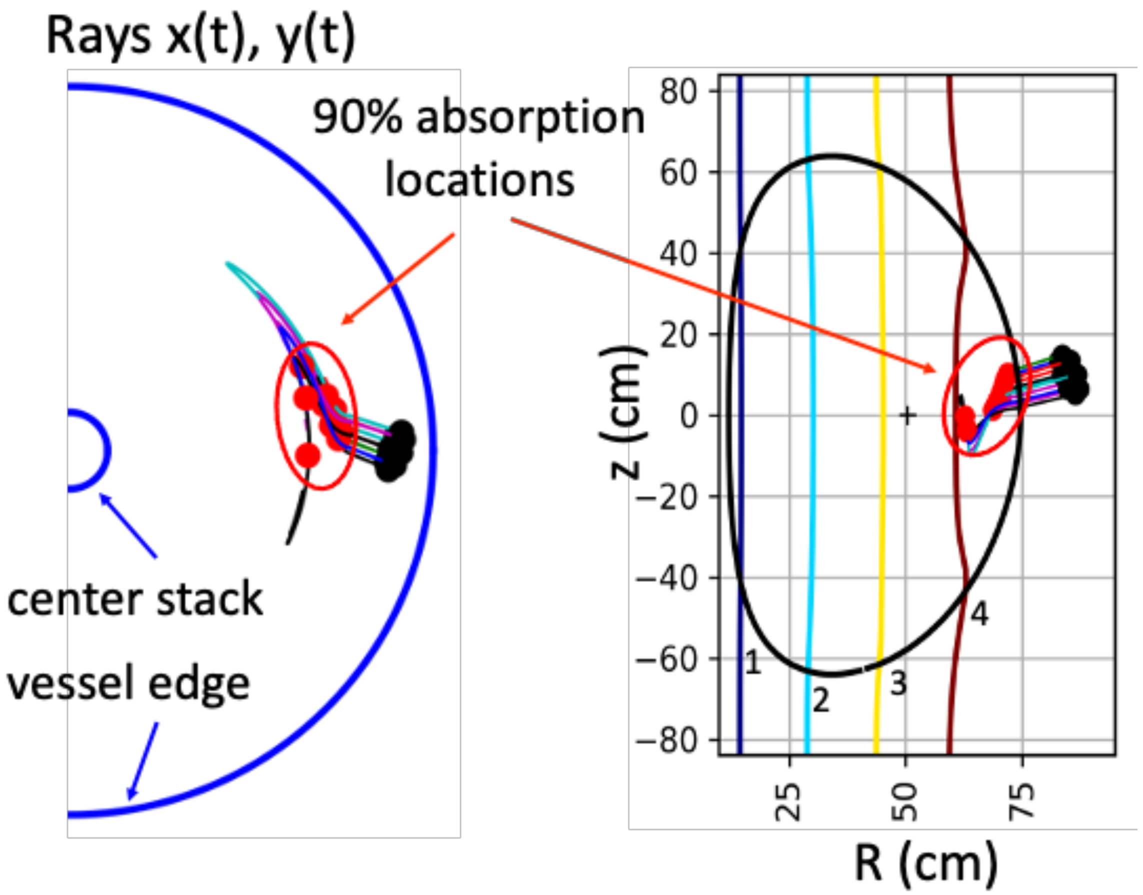
Plasma response not included, in these calculations

C. Schaefer, et. al, Physics of Plasmas 33 (2025)

Initial EBW program seeks to explore synergies

- Relative low B_T, high n_e of STs necessitates use of EBWs for fundamental absorption
- EBW heating: synergistically enhance LHI induced I_p current by lowering resistivity
  - 250 kW, 28 GHz
- T_e increases compatibility with non-inductive sustainment (i.e. neutral beam current drive)

B_T=0.4 T



Collaboration between ORNL and UW-Madison

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Plasma heating and current drive required for high performance fusion systems

- Burning plasma in fusion power plants will produce most of required heating power via alpha particles produced by fusion events

- Burning plasma will require auxiliary heating power:
  - initiate plasma
  - current ramp-up

- Plasma heating needed to reach ignition and provide
  - Control of MHD instabilities
  - Control against impurity events
  - Control plasma profiles

- Provide current drive for long-pulse or steady state operation

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Waves in plasmas

- Appleton-Hartree formula: general dispersion relation for O-mode waves, X-mode waves, and waves in unmagnetized plasmas

$$N^2 = 1 - \frac{X(1-X)}{1 - X - \frac{1}{2}Y^2 sin^2\theta \pm \left[\left(\frac{1}{2}Y^2 sin^2\theta\right)^2 + (1-X)^2 Y^2 cos^2\theta\right]}$$

where index of refraction $\mathrm{N} = \frac{kc}{\omega}$, $X = \frac{\omega_p^2}{\omega^2}$, $\mathrm{Y} = \frac{\omega_c}{\omega}$,

plasma frequency $\omega_p^2 = \frac{ne^2}{m\epsilon_0}$, cyclotron frequency $\omega_c^2 = \frac{e^2B^2}{m^2}$

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Dispersion relations

- Parallel propagation $N^2 = 1 - \frac{X}{1 \pm Y}$
- O-mode: $N^2 = 1 - X$
- X-mode: $N^2 = \frac{X(1-X)}{1-X-Y^2}$
  - Resonances at

    Upper hybrid frequency $\omega^2 = \omega_{pe}^2 + \omega_{ce}^2$

    Lower hybrid frequency $\omega^2 = \omega_{ce}\omega_{ci}\left(\frac{\omega_{pe}^2+\omega_{ce}\omega_{ci}}{\omega_{pe}^2+\omega_{ce}^2}\right)$

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EBW dispersion relation derived from perpendicular propagation dielectric permittivity tensor

- Dielectric permittivity tensor ($\boldsymbol{K}$) for waves in warm magnetized plasma found from Maxwell's Eq and a linearized Vlasov eq.

$$K_{ij} = \delta ij + \sum_s \frac{\omega_{ps}^2}{\omega} \left(\frac{m_s}{2T_s}\right)^{\frac{1}{2}} \frac{e^{-\lambda_s}}{k_z} \sum_{n=-\infty}^{\infty} T_{ij}$$

$$\mathbf{T} = \begin{pmatrix} n^2 I_n \frac{Z}{\lambda_s} & in(I_n' - I_n)Z & -nI_n \frac{Z'}{\sqrt{2\lambda_s}} \\ -in(I_n' - I_n)Z & \left(n^2 \frac{I_n}{\lambda_s} + 2\lambda_s I_n - 2\lambda_s I_n'\right) Z & i\sqrt{\lambda_s}(I_n' - I_n)\frac{Z'}{\sqrt{2}} \\ -nI_n \frac{Z'}{\sqrt{2\lambda_s}} & -i\sqrt{\lambda_s}(I_n' - I_n)\frac{Z'}{\sqrt{2}} & -I_n Z' \xi_n \end{pmatrix}$$

$$\lambda_s = \frac{T_s k_\perp^2}{m_s \Omega_s^2}$$

$$\xi_n = \frac{\omega - n\Omega_s}{k_z} \sqrt{\frac{m_s}{2T_s}}$$

$$Z(\zeta) = \sqrt{\pi} \int_{-\infty}^{\infty} \frac{e^{-t^2}}{t - \zeta} dt$$

- Dispersion relations found from $\boldsymbol{K}$ using

$$\mathbf{M} \cdot \mathbf{E} = 0 \text{ where } \mathbf{M} = \vec{k}\vec{k} - k^2 \mathbf{I} + \frac{\omega^2}{c^2} \mathbf{K}$$

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EBW dispersion relation derived from perpendicular propagation dielectric permittivity tensor

- Simplified for perpendicular propagation ($k_z \to 0$)

$$M_{13} = M_{23} = M_{31} = M_{32} = 0$$

$$M_{11} = 1 - \sum_s \frac{\omega_{ps}^2}{\omega} \frac{e^{-\lambda_s}}{\lambda_s} \sum_{n=-\infty}^{\infty} \frac{n^2 I_n(\lambda_s)}{\omega - n\Omega_s}$$

$$M_{12} = -M_{21} = -i \sum_s \frac{\omega_{ps}^2}{\omega} e^{-\lambda_s} \sum_{n=-\infty}^{\infty} \frac{n(I_n'(\lambda_s) - I_n(\lambda_s))}{\omega - n\Omega_s}$$

$$M_{22} = 1 - \frac{k_\perp^2 c^2}{\omega^2} - \sum_s \frac{\omega_{ps}^2}{\omega} \frac{e^{-\lambda_s}}{\lambda_s} \sum_{n=-\infty}^{\infty} \frac{n^2 I_n(\lambda_s) + 2\lambda_s^2 I_n(\lambda_s) - 2\lambda_s^2 I_n'(\lambda_s)}{\omega - n\Omega_s}$$

$$M_{33} = -\frac{k_\perp^2 c^2}{\omega^2} - \sum_s \frac{\omega_{ps}^2}{\omega} e^{-\lambda_s} \sum_{n=-\infty}^{\infty} \frac{I_n(\lambda_s)}{\omega - n\Omega_s}$$

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